Modern cosmology with x-ray luminous clusters of galaxies, Monday Lecture: Basic Cluste Cosmolog

at XXIX Heidelberg Graduate Days

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Modern cosmology with X-ray luminous clusters of galaxies

Monday Lecture: Basic Cluster Cosmology

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Outline:
Initial plan for the week
(that will somewhat depend on the interaction with the class)

Monday Lecture: Introduction to cosmology, cluster cosmology and the gas mass fraction experiment

Tuesday Lecture/Practice: CAMB and CosmoMC (download them at http://cosmologist.info/cosmomc/ and the fgas module at http://www.slac.stanford.edu/~drapetti/fgas_module/) including initial practice with SNe, and fgas.

Wednesday Lecture: Cluster abundance experiment

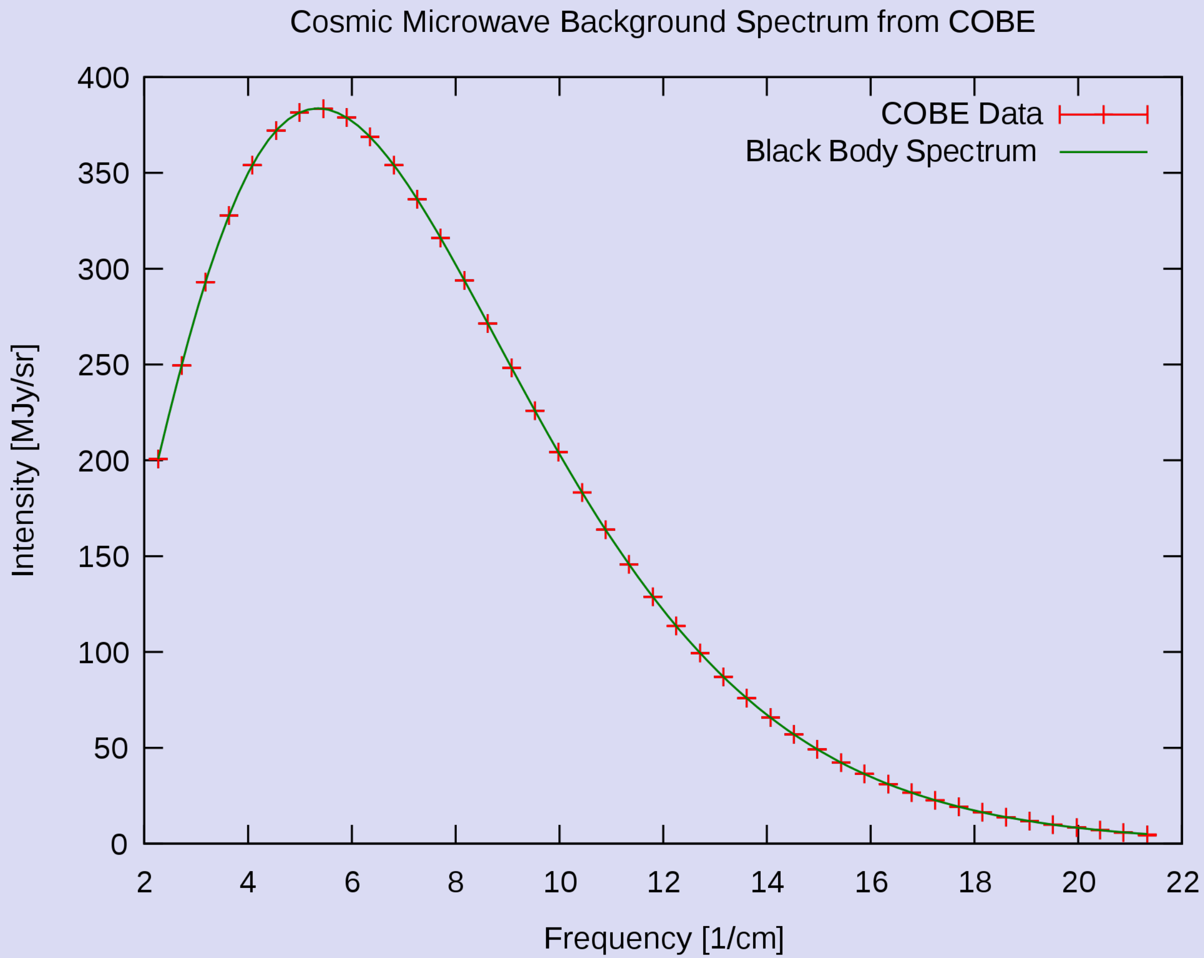
Thursday Lecture: Cosmological models and modeling

Friday Lecture/Practice: CosmoMC project: constraining a theoretical model

Recent discoveries and current results

Cosmic Microwave Background (CMB)

Nobel Prize in Physics 1978: Arno Penzias & Robert Wilson (CMB discovery in 1965) [Pyotr Kapitsa (Low-temperature physics)]
Nobel Prize in Physics 2006: John Mather & George Smoot (CMB blackbody and anisotropy)



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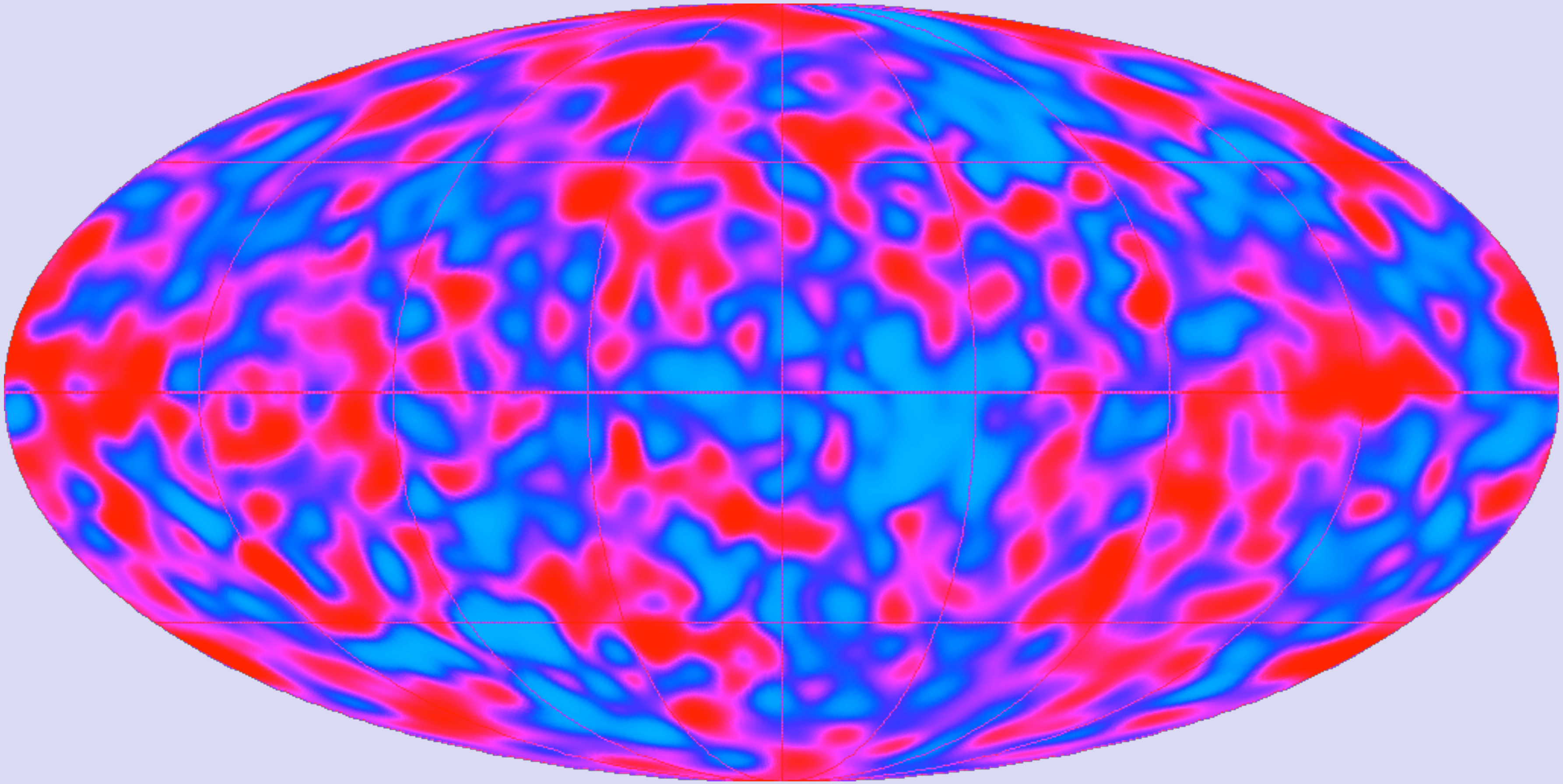
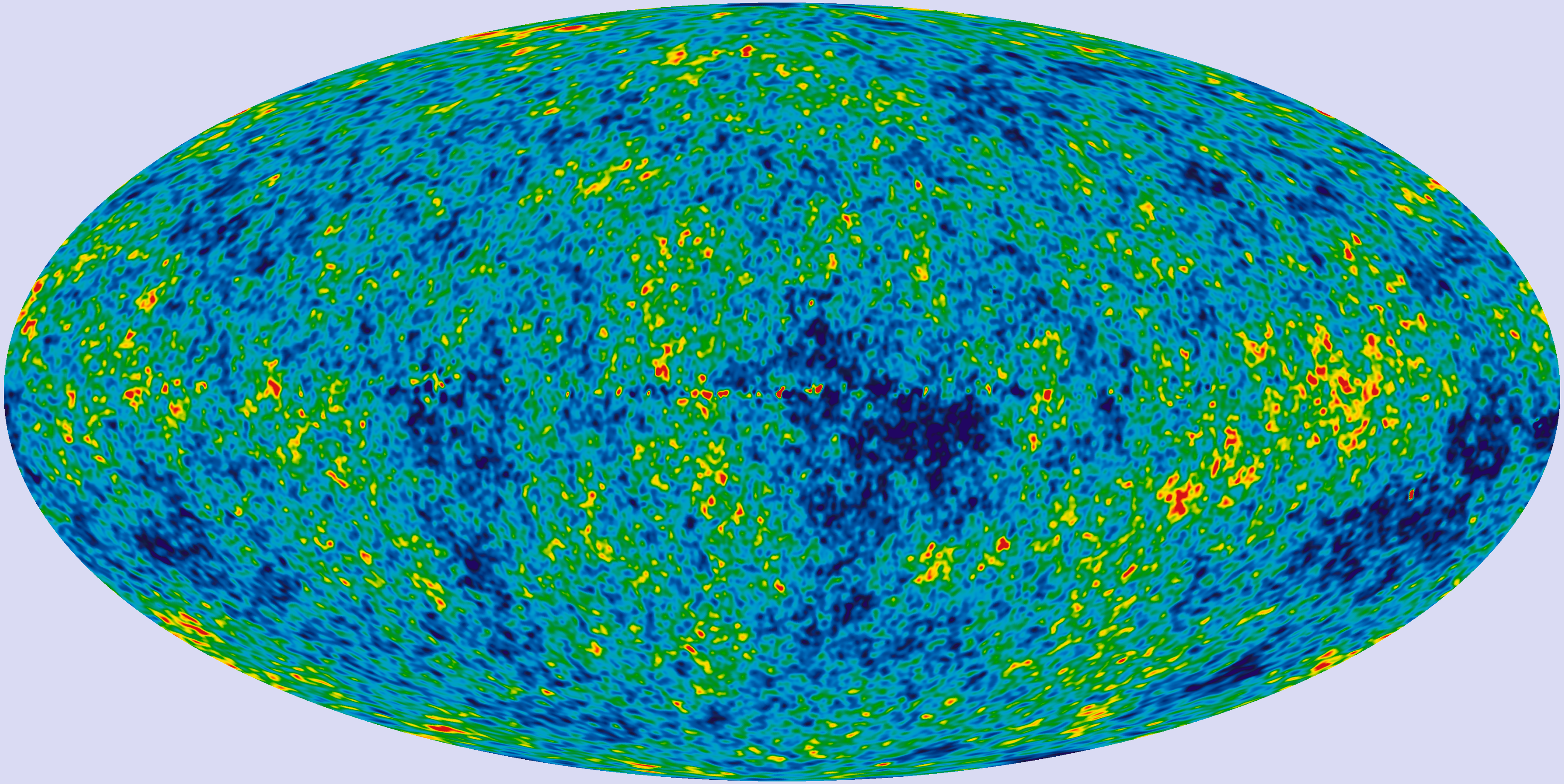
From NASA's COBE satellite



Cosmic Microwave Background (CMB)

Current measurements from NASA's WMAP satellite



Next: results from ESA's Planck satellite are coming next year…

Discovery of cosmic acceleration

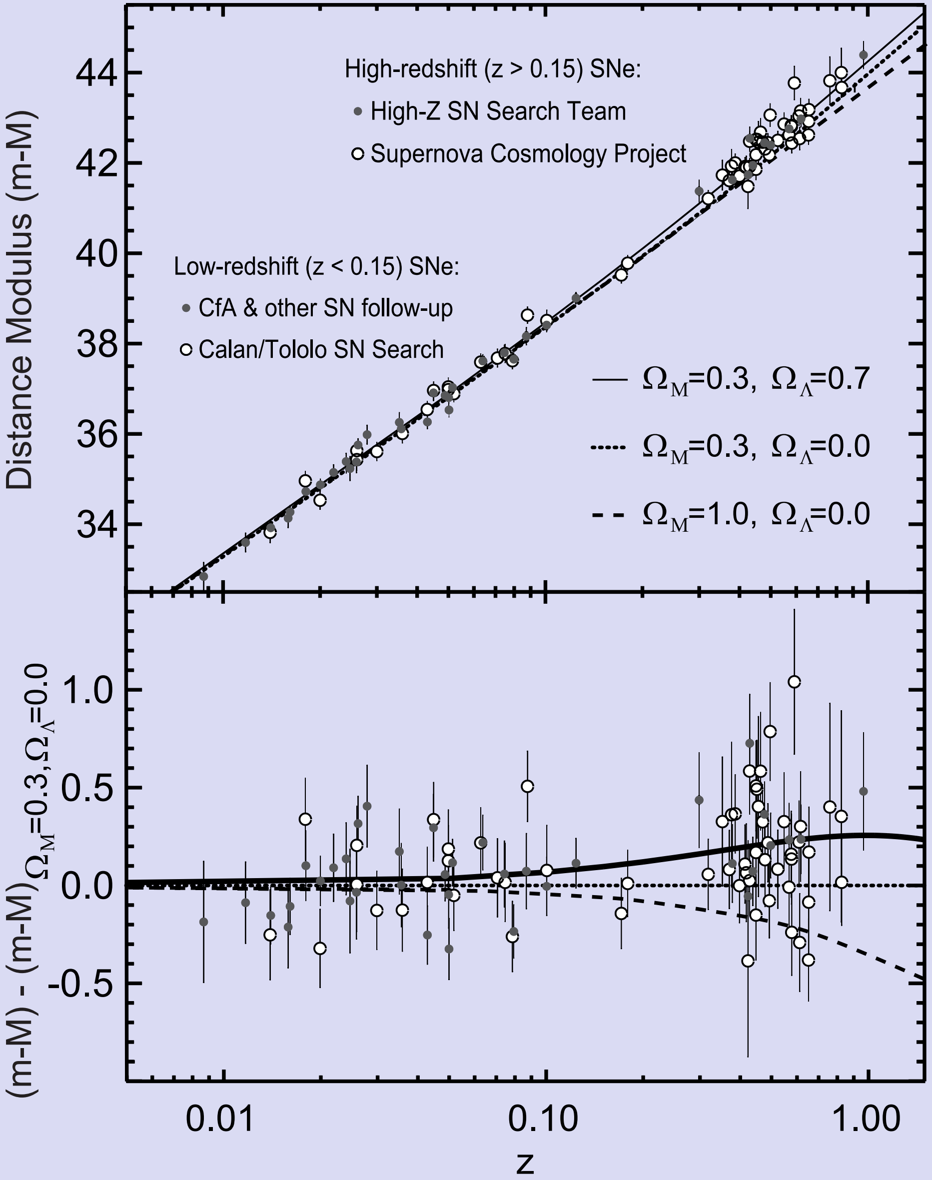
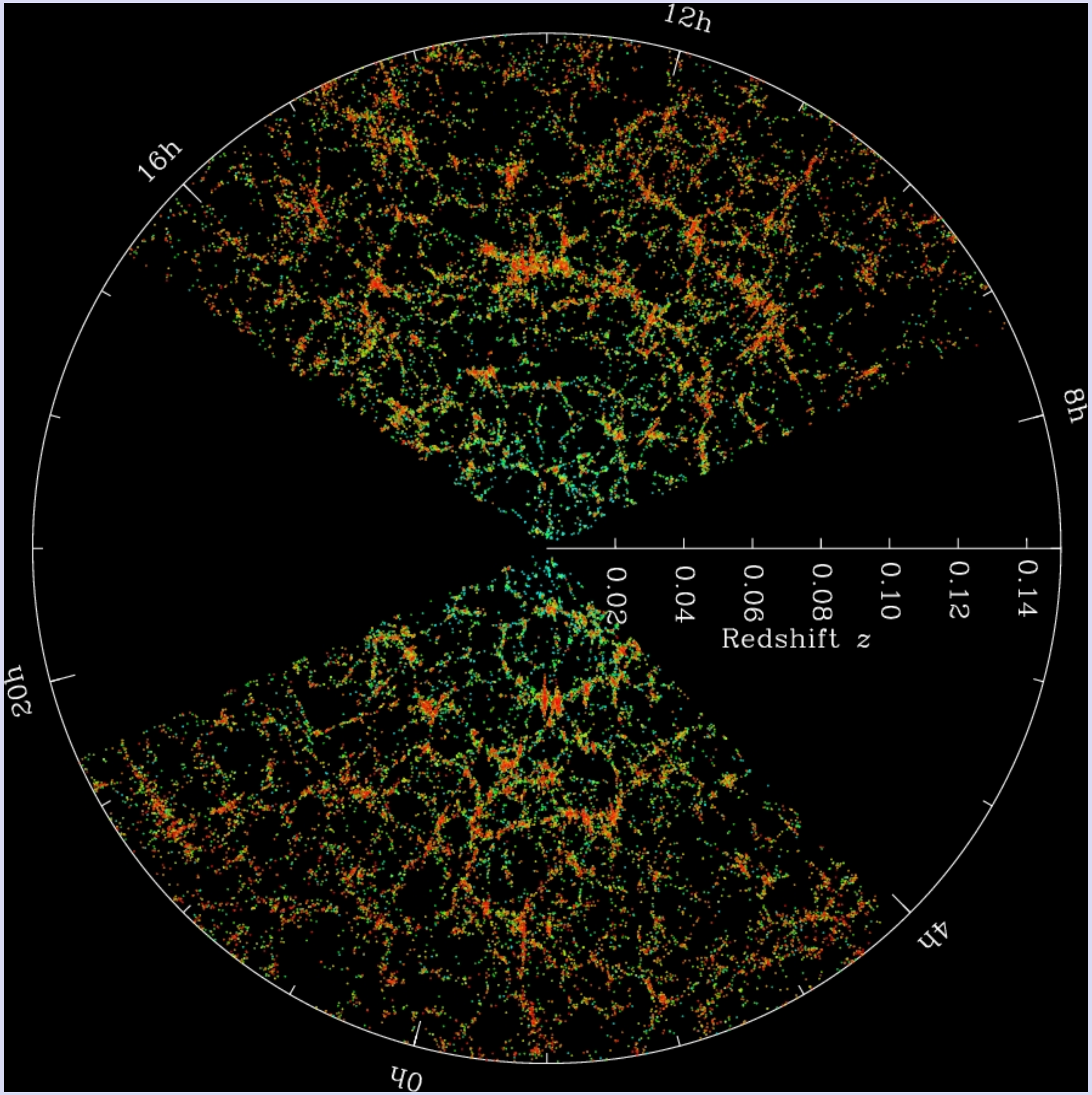


Figure from the dark energy review of Frieman, Turner & Huterer, 2008, ARA&A., 46, 385

-High-Z SN Search Team (HZT): Riess et al 1998
-Supernova Cosmology Project (SCP): Perlmutter et al et al. 1999

Nobel Prize Award in Physics 2011:
Saul Perlmutter (SCP)
Brian Schmidt & Adam Riess (HZT)

Large scale distribution of galaxies



Sloan Digital Sky Survey
(from the SDSS website)

Slice of a 3D map of galaxies

Galaxies are colored according to the ages of their stars: redder, more strongly cluster made of older starts.

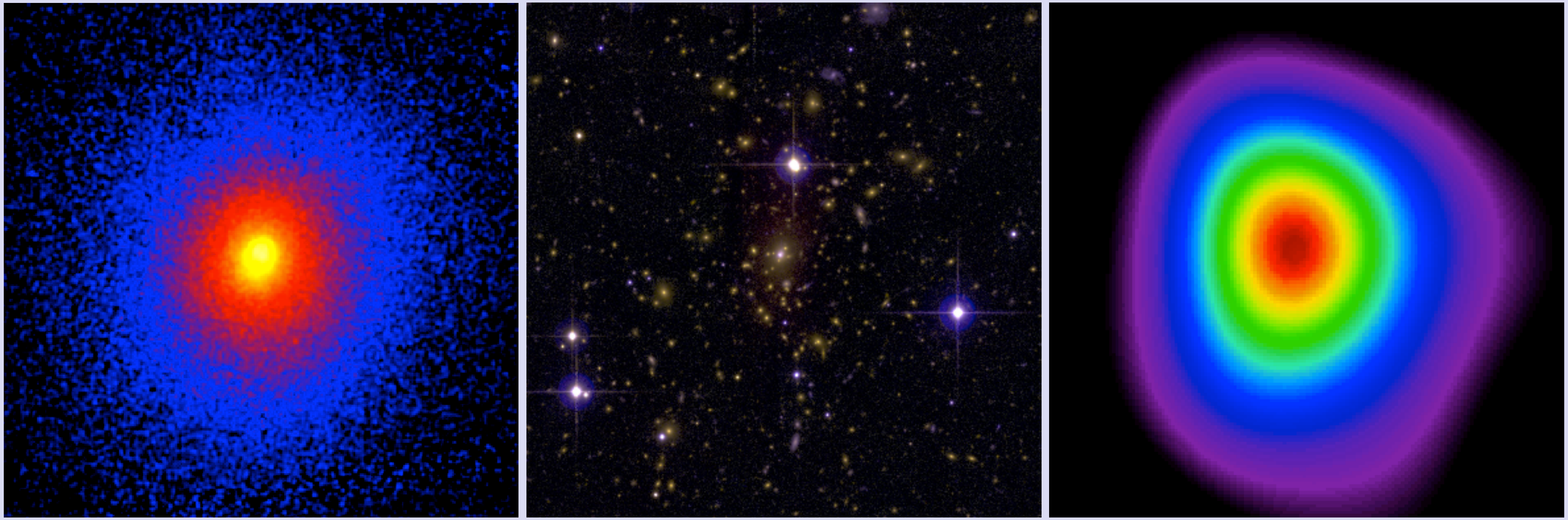
Outer circle: two billion light years

>930000 galaxies

Cluster cosmology

Cluster cosmology

Figure from Allen, Evrard & Mantz 11 (credits X-ray/Mantz; Optical/von der Linden et al; SZ/Marrone)

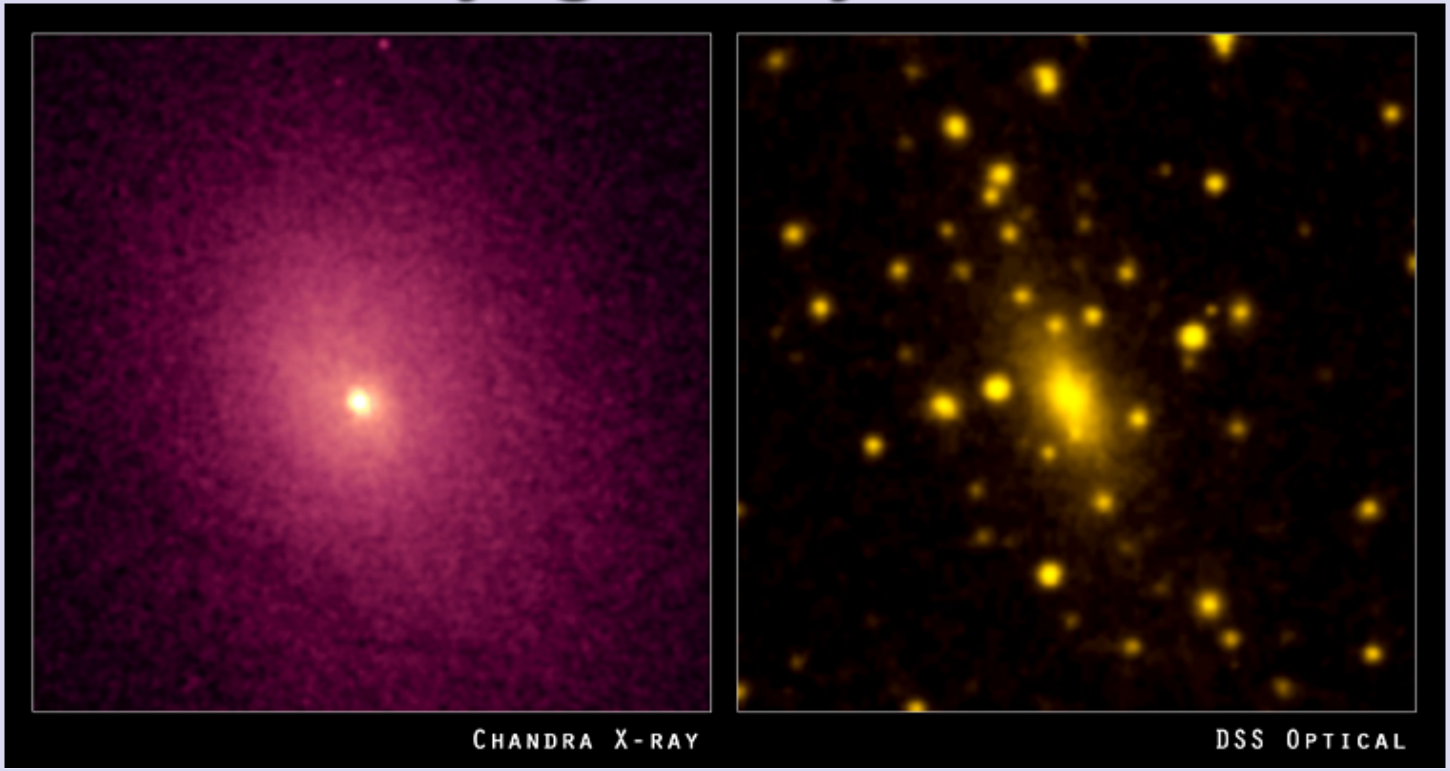


X-ray | Optical | Millimeter (SZ)

- Images of galaxy cluster Abell 1835 in different wavelength
- Cosmology with galaxy clusters using X-ray observations:
  - Gas mass fraction
  - Abundance of clusters and their observable-mass relations

Cluster cosmology review: Allen, Evrard & Mantz, 2011, ARA&A, 49, 409

X-ray galaxy cluster

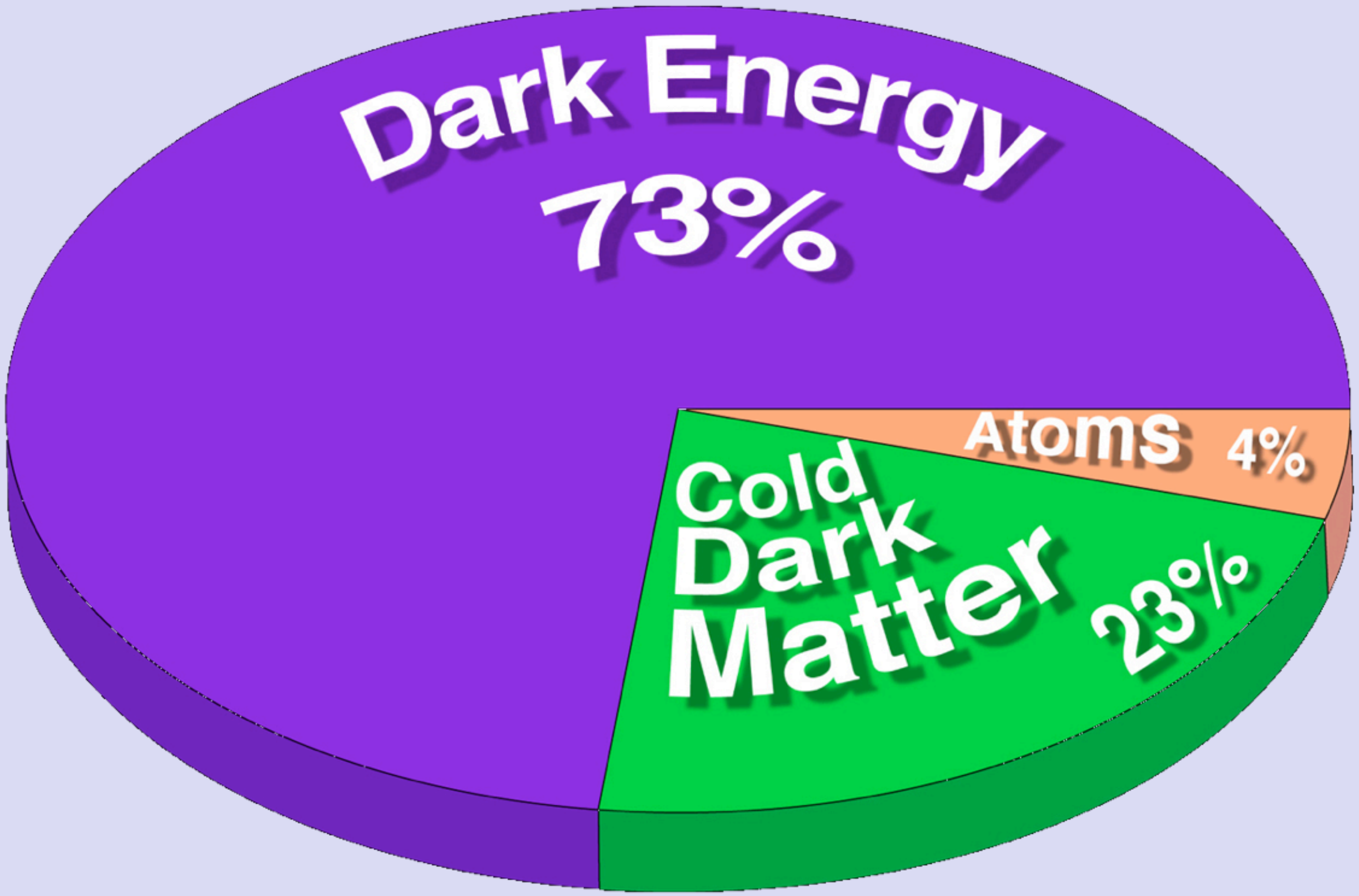


Galaxy cluster Abell 2029:
Thousands of galaxies optical (right panel)
Hot (multimillion Kelvin degrees) gas (left panel).
Dark matter (only gravitational interaction) >10^{15} solar masses.
In 1933 Fritz Zwicky proposed "missing matter" (dark matter) in clusters of galaxies
(Note: Central enormous elliptically shaped galaxy.)

Basic cosmology

Peebles; Luchin & Mataresse; Peacock; Dodelson; Weinberg ; Mukhanov; etc.

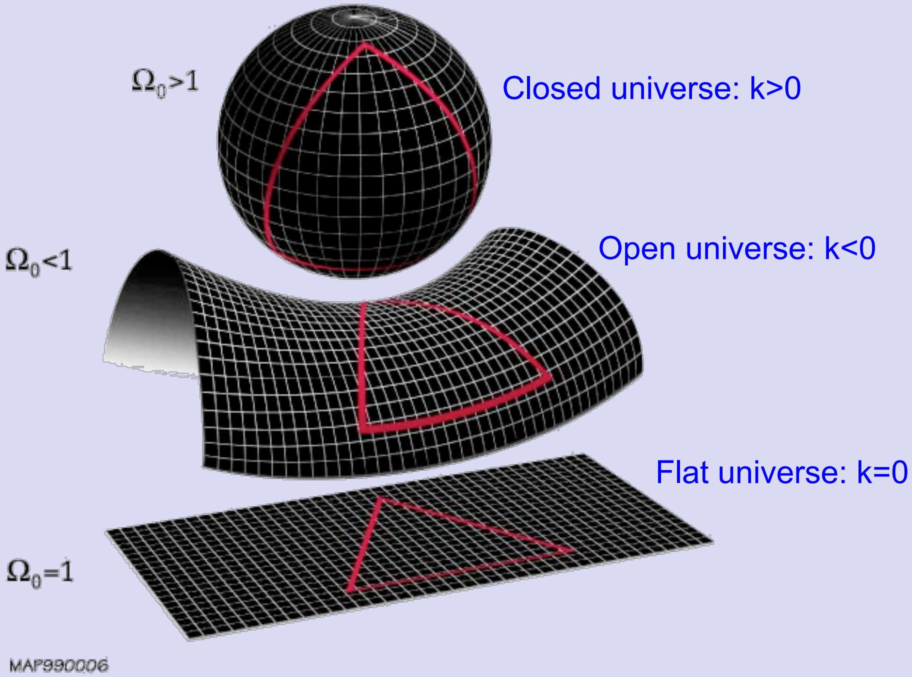
Cosmic pie



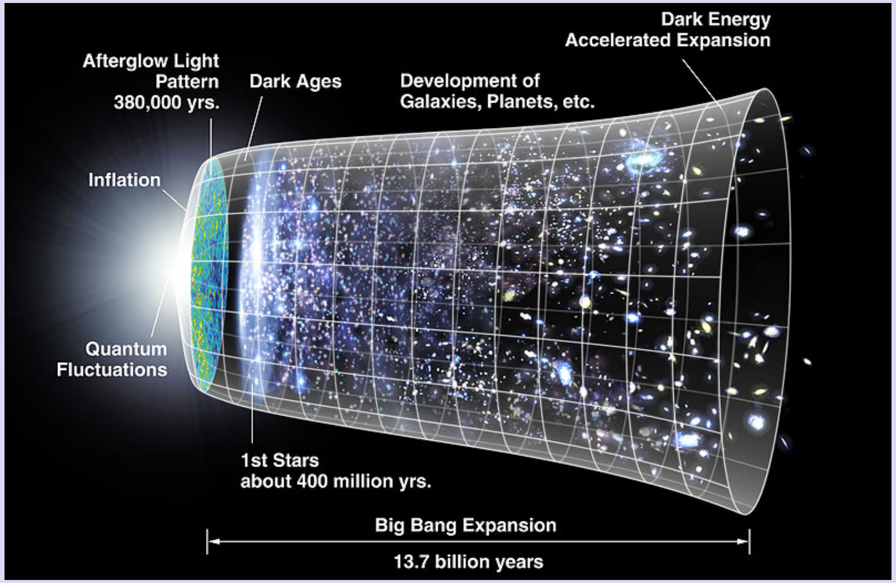
Space-time metric of the Universe

Friedmann-Lemaitre-Robertson-Walker (FLRW) metric: for an isotropic and homogenous space-time (current data indicate that at large scale these assumptions are nearly valid)

$$ds^2 = dt^2 - a^2(t)\left[dr^2/(1-kr^2) + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$



Expansion history a(t)



Gravity and energy density

Einstein's equations

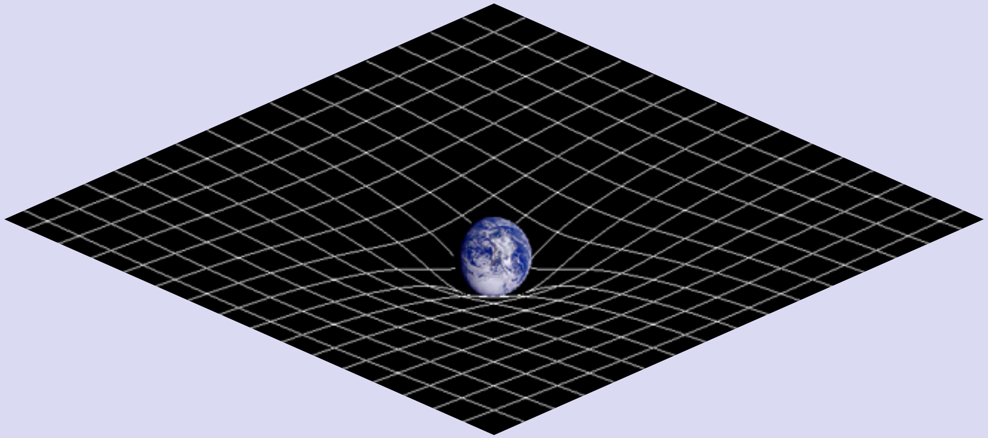
$$G^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi T^{\mu\nu}$$

$$c = G = 1$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}$$

$R^{\mu\nu}$ Ricci tensor
R Ricci scalar
$G^{\mu\nu}$ metric tensor

Interaction between gravity and energy-matter



John Wheeler: Matter tells space how to curve and space tells matter how to move

Useful reference for perturbation theory: Ma & Bertschinger 1995, ApJ, 455, 7

Cosmic energy content and expansion

Example of stress-energy tensor:

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Fluid in a thermodynamic equilibrium

$$\begin{aligned} H^2 = \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G\rho}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \end{aligned}$$

Friedmann equations: key equations in cosmology; Einstein field equations for the FRWL metric

$$\Lambda = 8\pi G \rho_{\mathrm{VAC}} = -8\pi G p_{\mathrm{VAC}}$$

Energy density of the vacuum

Dark energy review: Frieman, Turner & Huterer 2008, ARA&A., 46, 385

Cosmic energy content and expansion

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

Friedmann equation
ρ sum of the energy densities of matter, dark energy, radiation

$$w = \frac{p_{de}}{\rho_{de}}$$

Dark energy equation of state

$$a(t) = \frac{1}{1+z}$$

a(t) scale factor
z redshift

$$E(a) = \left[\Omega_\mathrm{m}\, a^{-3} + \Omega_\mathrm{de}\, a^{-3(1+w)} + \Omega_\mathrm{k}\, a^{-2}\right]^{1/2}$$

$$E(z) = \sqrt{\Omega_\mathrm{m}(1+z)^3 + \Omega_\mathrm{de} f(z) + \Omega_\mathrm{k}(1+z)^2}$$

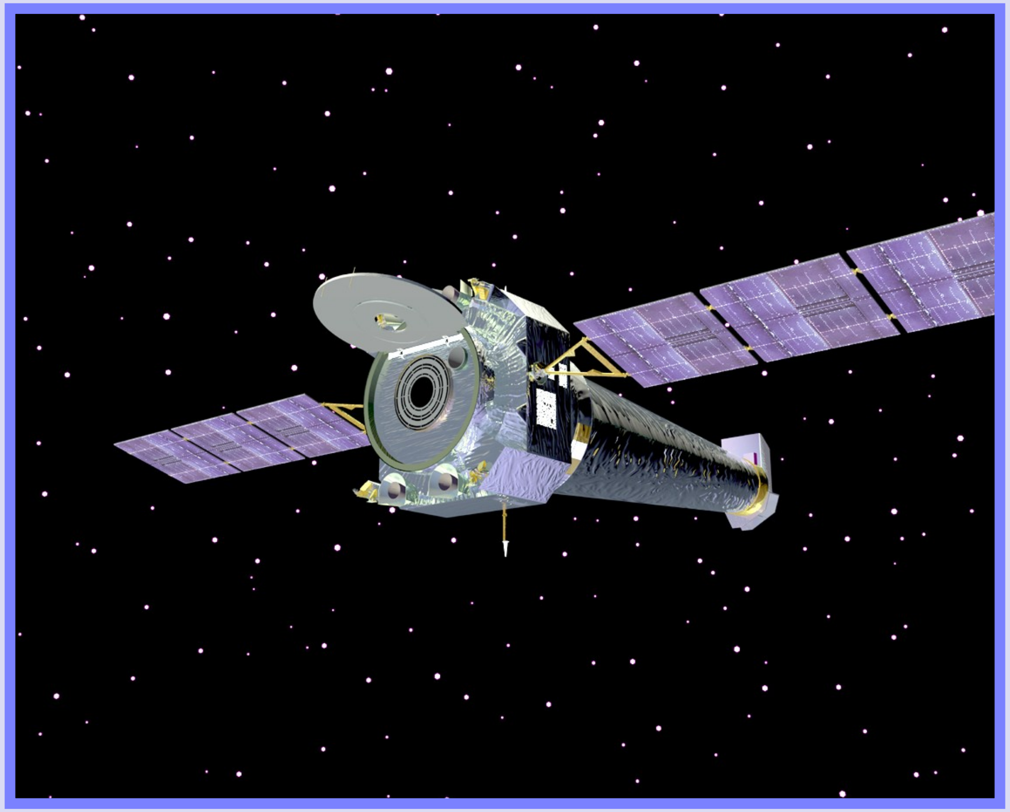
Evolution parameter
$E(a)=H(a)/H_0$

i) flat ΛCDM w=-1, Ω_k=0
ii) flat *w*CDM w constant, Ω_k=0
iii) non-flat ΛCDM w=-1, Ω_k constant

The $f_{gas}(z)$ experiment

e.g. Allen et al 02, 04, 08; Ettori et al 03, 09;
Rapetti et al. 05, 07, 08; LaRoque et al 06

Chandra X-ray Observatory



Cluster cosmology revolutionized

First opportunity to carry out:

->Detailed spatially-resolved and

->X-ray spectroscopy of galaxy clusters.

Technical details for ACIS instrument (X-ray CCDs):

• Field of view 16x16 $arcmin^2$
• Good spectral resolution ~100eV over 0.5-8 keV range.
• Exquisite spatial resolution (0.5 arcsec FWHM).

Measuring comic matter content using the gas mass fraction of X-ray luminous galaxy clusters

Consider a spherical region of observed angular radius θ within which the gas mass fraction is measured

$R = \theta\, d_A$ R physical size

$L_x = 4\pi\, d_L^2 F_x$ L_x X-ray Luminosity of the region
F_x detected flux

$d_L = d_A (1+z)^2$ d_L luminosity distance
d_A angular diameter distance

Since the X-ray emission is mainly due to collisional processes (bremsstrahlung and emission line) and is optically thin

$L_x \propto n^2 V$ n mean matter density of colliding gas particles
V volume of the emitting region: $V = 4\pi(\theta d_A)^3 / 3$

Measuring comic matter content using the gas mass fraction of X-ray luminous galaxy clusters

$$n \propto \frac{d_L}{d_A^{3/2}} \longrightarrow M_{gas} \propto nV \propto d_L d_A^{3/2}$$

M_{gas} observed gas mass within the measurement radius

$$M_{tot} \propto d_A$$

M_{tot} total mass determined by X-ray data assuming hydrostatic equilibrium

M_{gas}	gas mass	$\propto d_A(z)^{2.5}$	(X-ray Luminosity)
M_{tot}	total cluster mass	$\propto d_A(z)$	(primarily X-ray Temperature)

$$f_{gas} = \frac{M_{gas}}{M_{tot}} \propto d_L d_A^{1/2}$$

f_{gas} gas mass fraction

Measuring comic matter content using the gas mass fraction of X-ray luminous galaxy clusters

$$f_{gas} = \frac{M_{gas}}{M_{tot}} \propto d_A(z)^{1.5}$$

$$s = f_{stars}/f_{gas} = (0.16 \pm 0.05)h_{70}^{0.5}$$ s baryonic mass fraction in stars

Lin & Mohr 04, Fukugita et al 98, White et al 93

$$f_{baryon} = f_{stars} + f_{gas} = f_{gas}(1+s)$$ Baryon mass fraction

Measuring comic matter content using the gas mass fraction of X-ray luminous galaxy clusters

$$f_{baryon} = b\frac{\Omega_b}{\Omega_m}$$

The matter content of rich clusters of galaxies is expected to provide an almost fair sample of the matter content of the Universe (White & Frenk 91, White et al. 93, Eke et al. 98).

b, the bias factor accounts for the relatively small amount of gas expelled when clusters form.

$$\Omega_m = \frac{b\Omega_b}{f_{gas}(1+s)}$$

+HST+BBNS priors when clusters alone or +CMB data

Measuring cosmic acceleration using the gas mass fraction of X-ray luminous galaxy clusters

$$f_{gas}^{ref}(z) = \frac{b(z)\gamma K}{1+s(z)}\left(\frac{\Omega_b}{\Omega_m}\right)\varepsilon(\theta)\left[\frac{d_A^{ref}(z)}{d_A^{\mathrm{mod}}(z;\theta)}\right]^{3/2}$$

Apparent evolution of the gas mass fraction

$$\varepsilon(\theta) = \left[\frac{H^{\mathrm{mod}}(z;\theta)\, d_A^{\mathrm{mod}}(z;\theta)}{H^{ref}(z)\, d_A^{ref}(z)}\right]^{\eta}$$

Small angular correction

$\eta = 0.214 \pm 0.022$ Measured from the data profiles

Measuring cosmic acceleration using the gas mass fraction of X-ray luminous galaxy clusters

Small angular correction that accounts for the angle subtended at the measurement radius r_{2500} as the underlying cosmology varies

$$\varepsilon(\theta) = \left(\frac{\theta_{2500}^{ref}}{\theta_{2500}^{\mathrm{mod}}} \right)^{\eta}$$

For each cluster the measured f_{gas} value at r_{2500} corresponds to a fixed angle θ_{2500}^{ref} for the reference cosmology that is slightly different from that $\theta_{2500}^{\mathrm{mod}}$ for the test cosmology.

$$M_{2500} \propto 4\pi r_{2500}^{3} \rho_{crit} / 3$$

Mass at the measurement radius r_{2500}, for which the density is 2500 the critical density

$$\rho_{crit} = 3H(z)^2 / 8\pi G$$

ρ_{crit} critical density

Measuring cosmic acceleration using the gas mass fraction of X-ray luminous galaxy clusters

Assuming hydrostatic equilibrium (HSE) in the intracluster medium (ICM) and spherical symmetry we can calculate the mass within a given radius using the following expression (Sarasin 1988)

$$M(r) = -\frac{rkT(r)}{G\mu m_p}\left[\frac{d\ln n}{d\ln r} + \frac{d\ln T}{d\ln r}\right]$$

n(r) is the gas density
T(r) ICM temperature
k Boltzmann constant
μm_p mean molecular weight

Measuring cluster masses is one of the cornerstones of cluster cosmology. Under those assumptions we can measure the total mass from density n(r) and temperature T(r) profiles obtained from X-ray data. Note also that M(r) depends more strongly on T(r) than n(r).

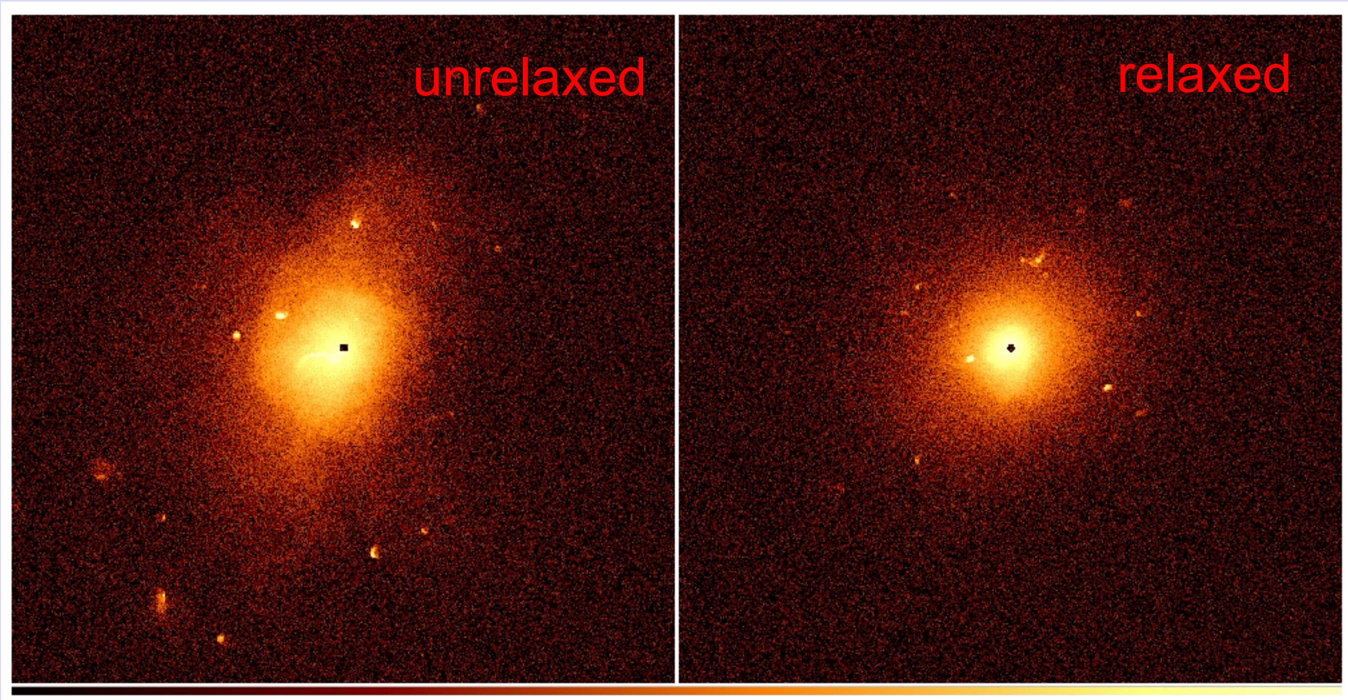
Given that the temperature, and temperature and density gradients, in the region of θ_{2500} are likely to be constant, we have

$$M_{2500} \propto r_{2500}$$

X-ray galaxy cluster cosmology:
How well can we measure cluster mass?

Hydro dynamical simulations of X-ray galaxy clusters

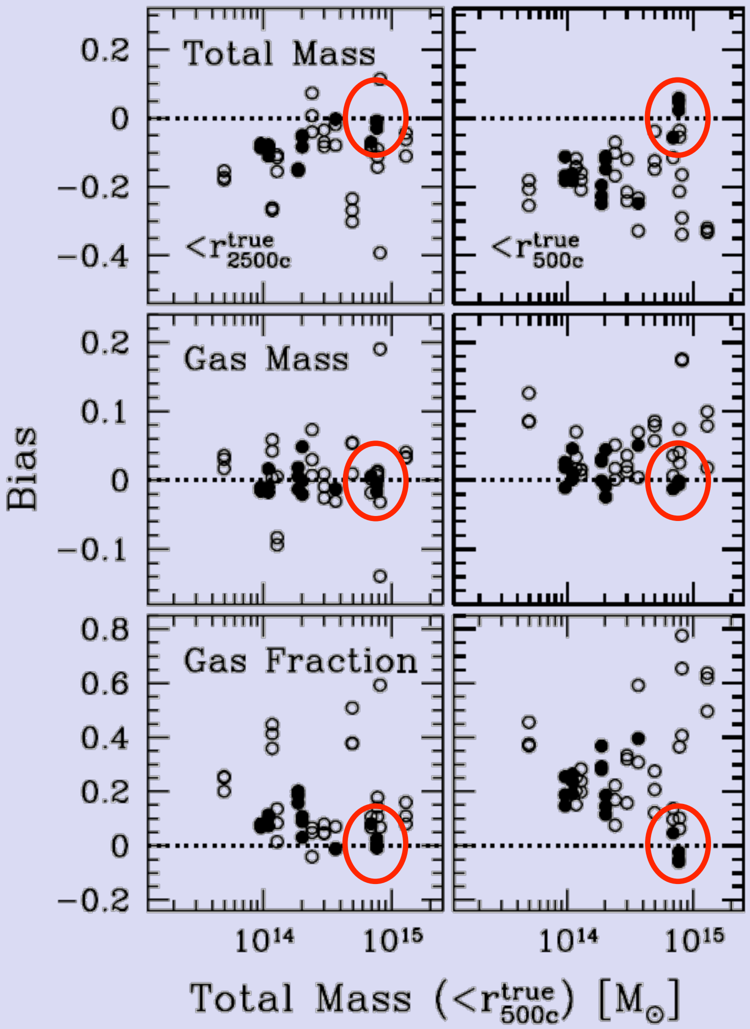
Nagai, Kravtsov, Vikhlinin 06



Very good news for X-ray galaxy cluster cosmology from the most recent simulations: systematics are relatively small and can be quantified.

How accurately can we measure the mass?

Nagai, Kravtsov, Vikhlinin 06



For the largest, hottest (kT>5keV), relaxed clusters (selection based on X-ray morphology) we currently expect to measure:

a) X-ray gas mass to ~1% accuracy.

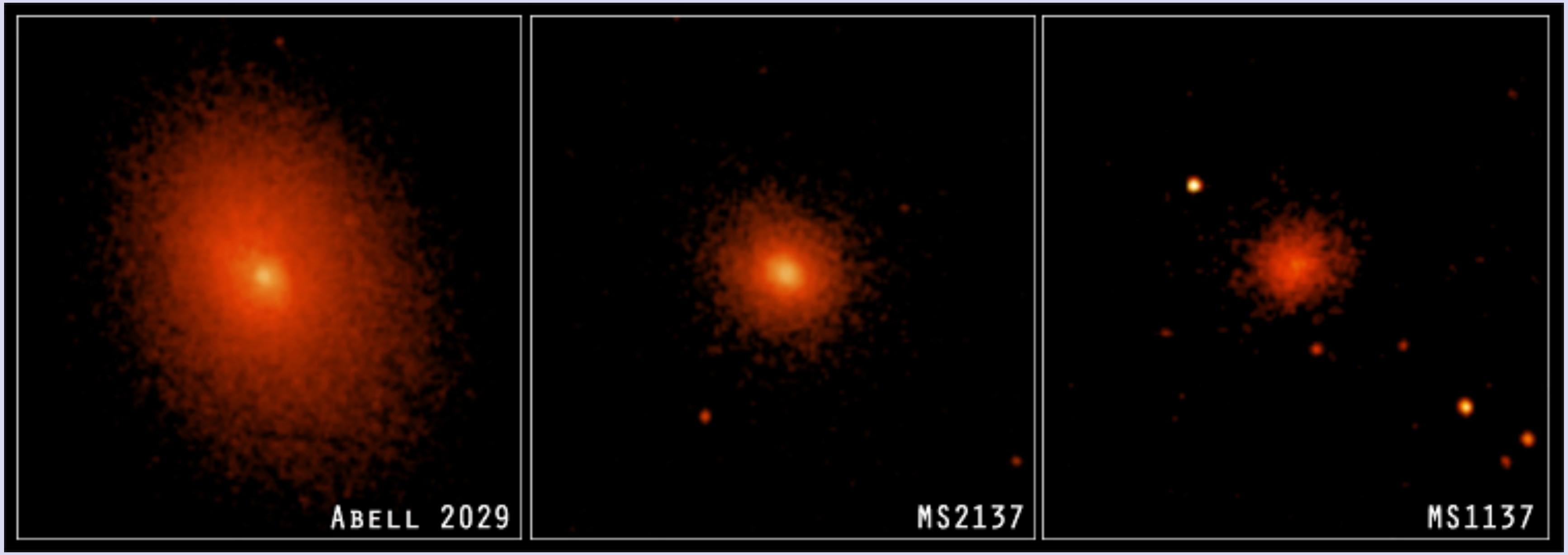
b) Total mass to few % accuracy (both bias and scatter).

Largest, relaxed clusters (filled points) inside red circles.

X-ray galaxy cluster data

It is crucial to use only dynamically relaxed clusters:

Regular X-ray morphology, Low ellipticities, Minimal centroid variation, Sharp central brightness peaks centered on their dominant elliptical galaxies.



Measuring cosmic acceleration using the gas mass fraction of X-ray luminous galaxy clusters

$$\left.\begin{aligned} &M_{2500} \propto r_{2500} \\ &M_{2500} \propto 4\pi r_{2500}^{3}\rho_{crit}/3 \\ &\rho_{crit} = 3H(z)^{2}/8\pi G \end{aligned}\right\} \quad r_{2500} \propto H(z)^{-1}$$

$$r_{2500} \propto H(z)^{-1} \longrightarrow \theta_{2500} = r_{2500}/d_A \propto [H(z)d_A]^{-1}$$

Angle spanned by r_{2500} at redshift z

$$\varepsilon(\theta) \equiv \frac{\varepsilon(\theta)^{\mathrm{mod}}}{\varepsilon(\theta)^{ref}} = \left(\frac{\theta_{2500}^{ref}}{\theta_{2500}^{\mathrm{mod}}}\right)^{\eta} \approx \left[\frac{H^{\mathrm{mod}}(z;\theta)\,d_A^{\mathrm{mod}}(z;\theta)}{H^{ref}(z)\,d_A^{ref}(z)}\right]^{\eta}$$

Measuring cosmic acceleration using the gas mass fraction of X-ray luminous galaxy clusters

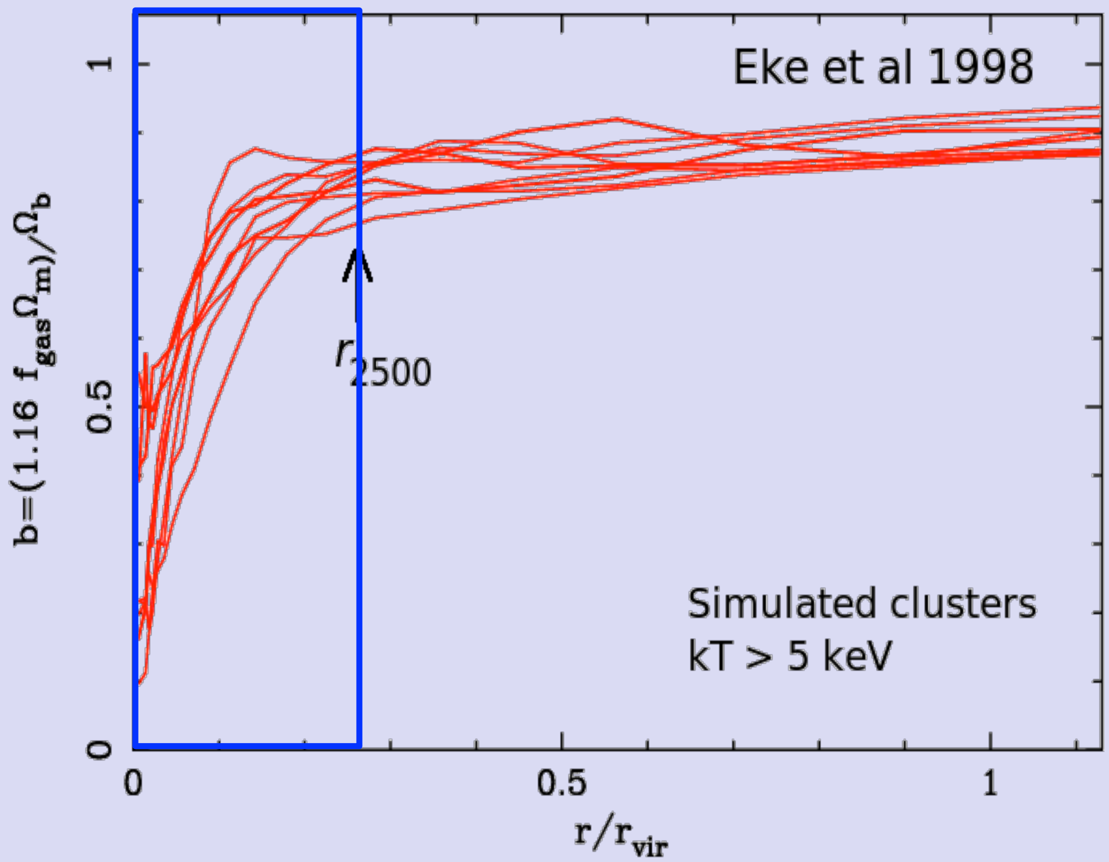
$$F(d_A^{\mathrm{mod}}) \equiv \frac{d_A^{\mathrm{mod}}(z;\theta)^{3/2}}{\varepsilon(\theta)^{\mathrm{mod}}} = \frac{b(z)\gamma K}{1+s(z)}\left(\frac{\Omega_b}{\Omega_m}\right)\left[\frac{d_A^{ref}(z)^{3/2}}{\varepsilon(\theta)^{ref} f_{gas}^{ref}(z)}\right]$$

Angular diameter distance measurement for the fgas experiment

Angular diameter distance measurement for the SNIa experiment: homework (for tomorrow)

Gas mass: simulations
Low scatter total mass proxy

ΛCDM cosmology

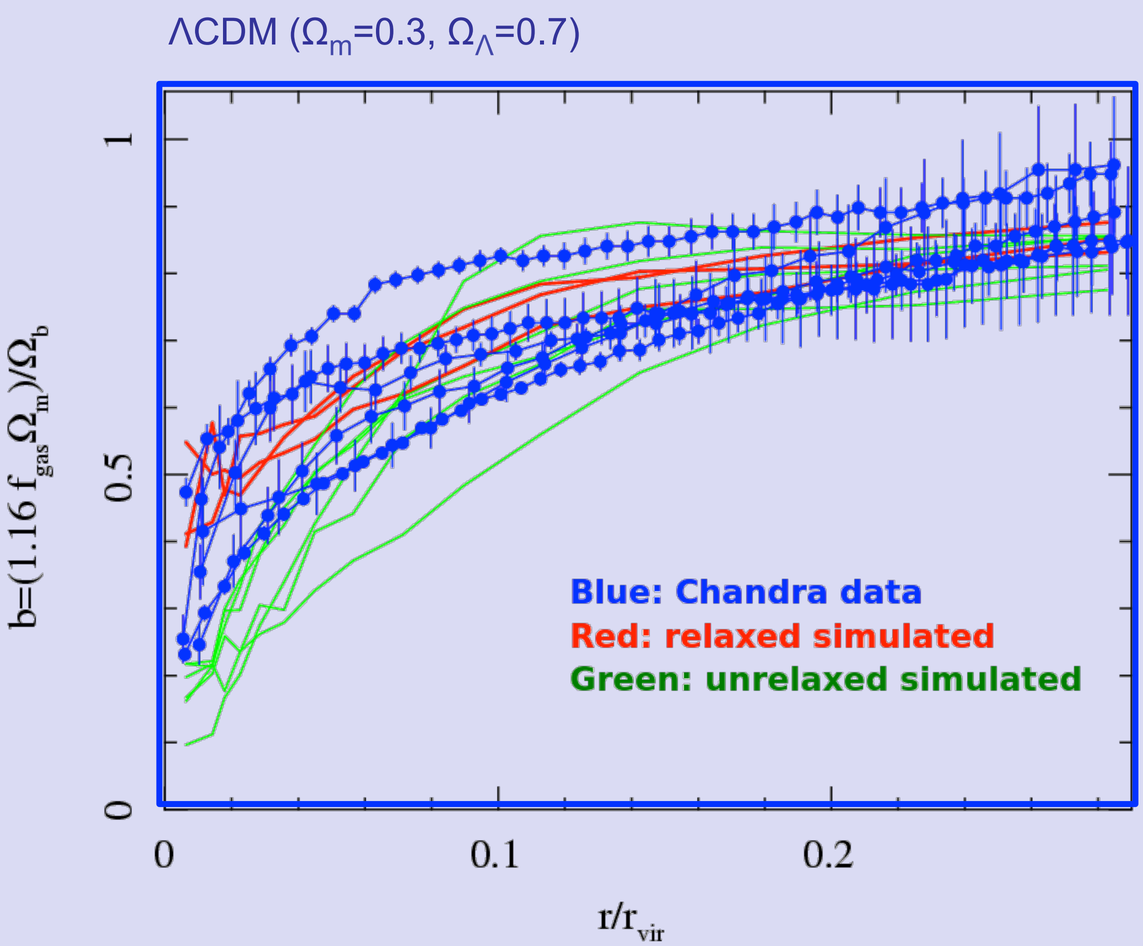


Simulations indicate low cluster-to-cluster scatter

Gas-mass low-scatter total-mass proxy through f_{gas}

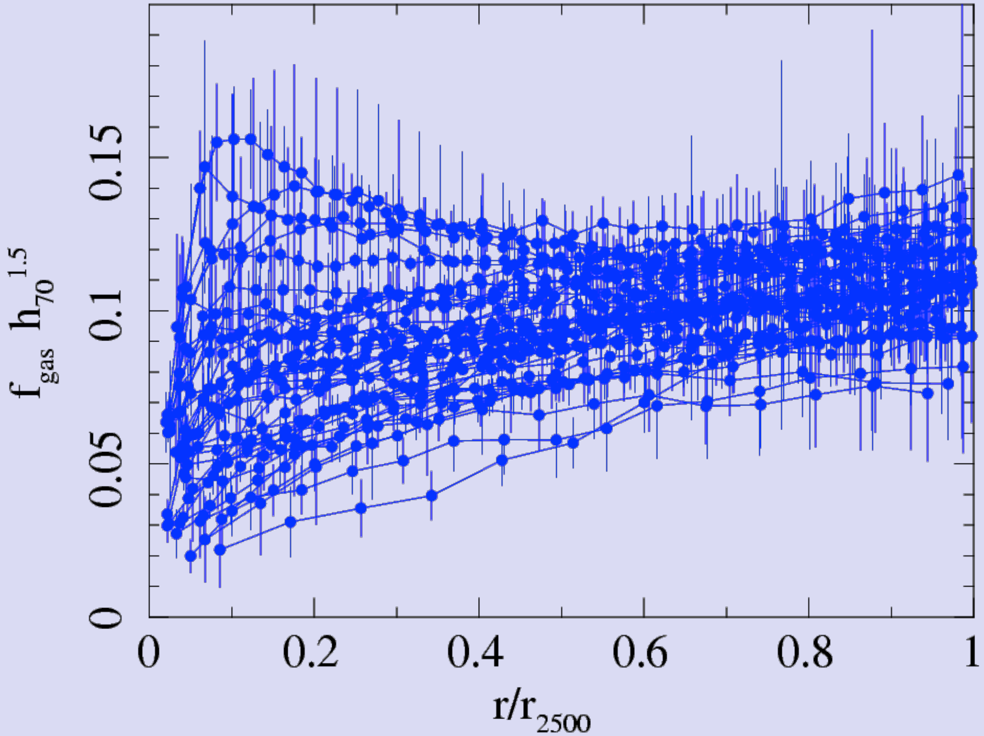
Simulations indicate that baryonic mass fraction in clusters is slightly lower than mean value for the Universe as a whole. Some gas is lifted beyond the virial radius by shocks (e.g. Evrard et al 90, Thomas & Couchman 92, Navarro & White 93; NFW 95 etc, Kay et al 04, Ettori et al 06, Crain et al 06, Nagai et al 07).

Gas mass: data
Low scatter total mass proxy



Undetected systematic scatter when weighted mean scatter ~5% in distance

Chandra fgas(r) data: 42 relaxed clusters with redshifts 0.06<z<1.07



Fitting a constant value at r_{2500} :

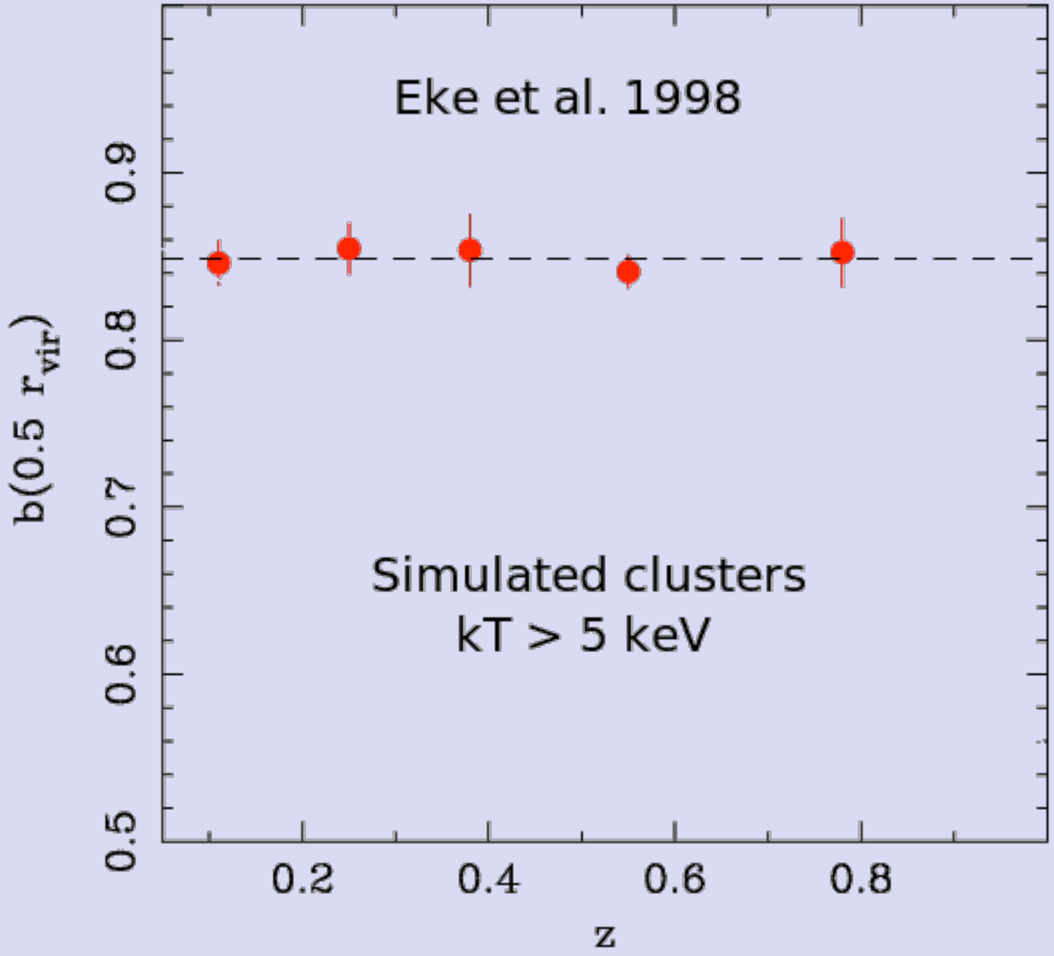
$f_{gas}(r_{2500})=(0.1104\pm0.0016)h_{70}^{-1.5}$

Fitting a power law $(0.7\text{-}1.2)r_{2500}$:

$f_{gas}(r_{2500})=(0.1105\pm0.0005)(r/r_{2500})^{0.214\pm0.022}$

Assuming hydrostatical equilibrium and spherical symmetry (only relaxed clusters).

Gas mass: simulations
Low scatter total mass proxy

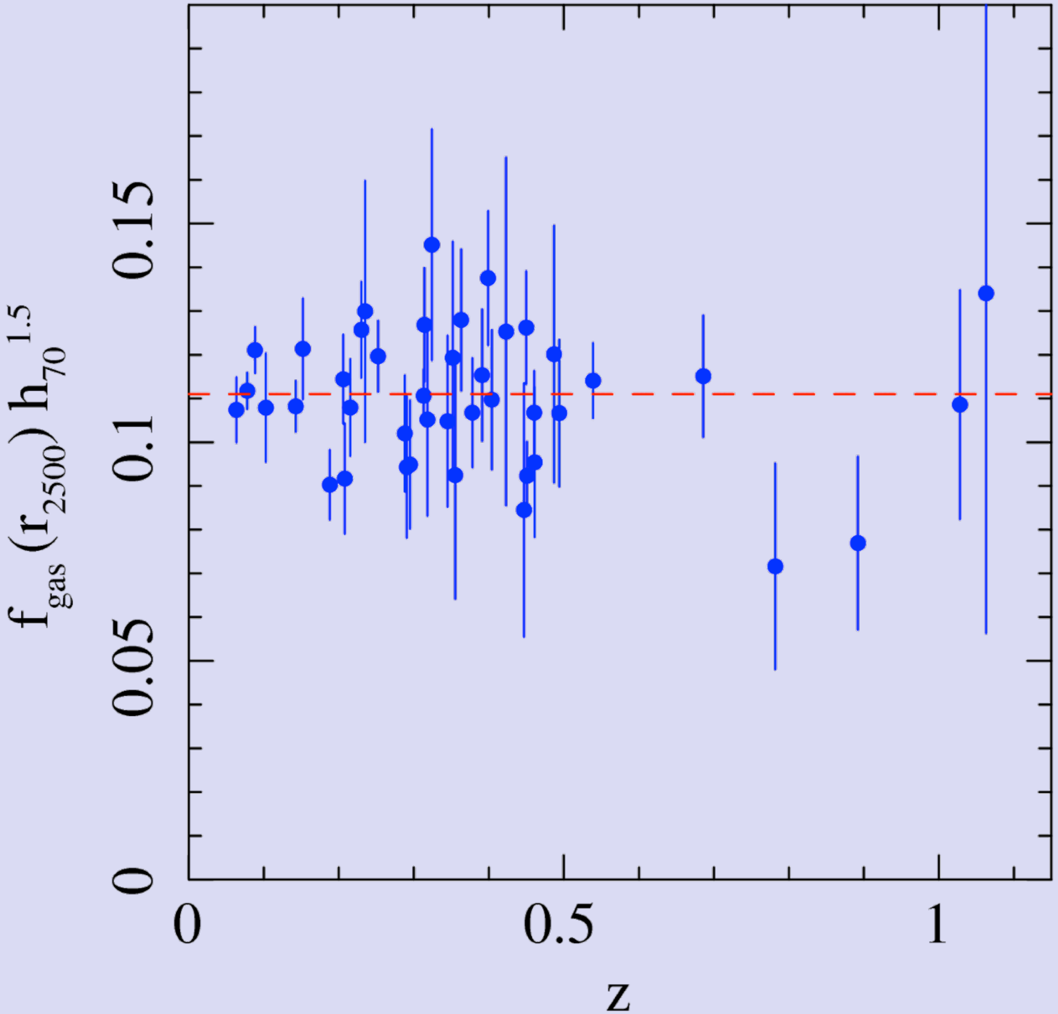


Tens of simulated clusters, $0<z<0.8$, $kT>5$keV

Minimal evolution at r_{2500}

Gas-mass low-scatter total-mass proxy through f_{gas}

Gas mass fraction: data
Low scatter total mass proxy



ΛCDM (Ω_m=0.3, Ω_Λ=0.7)

42 dynamically relax clusters

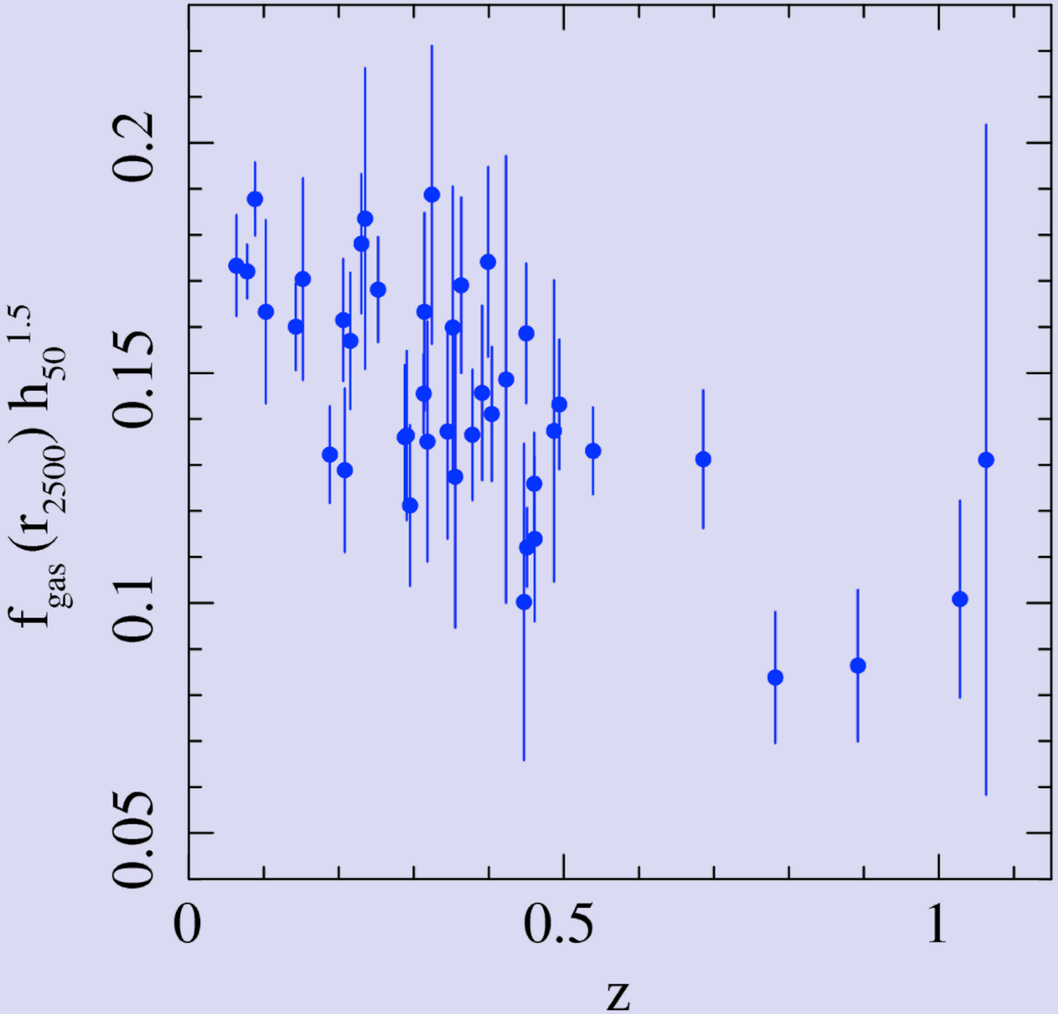
Hot kT>5keV

0.06<z<1.07

Scatter <~10% in f_{gas}

Undetected systematic scatter when weighted mean scatter ~5% in distance

Gas mass fraction: data
Low scatter total mass proxy



SCDM (Ω_m=1.0, Ω_Λ=0.0)

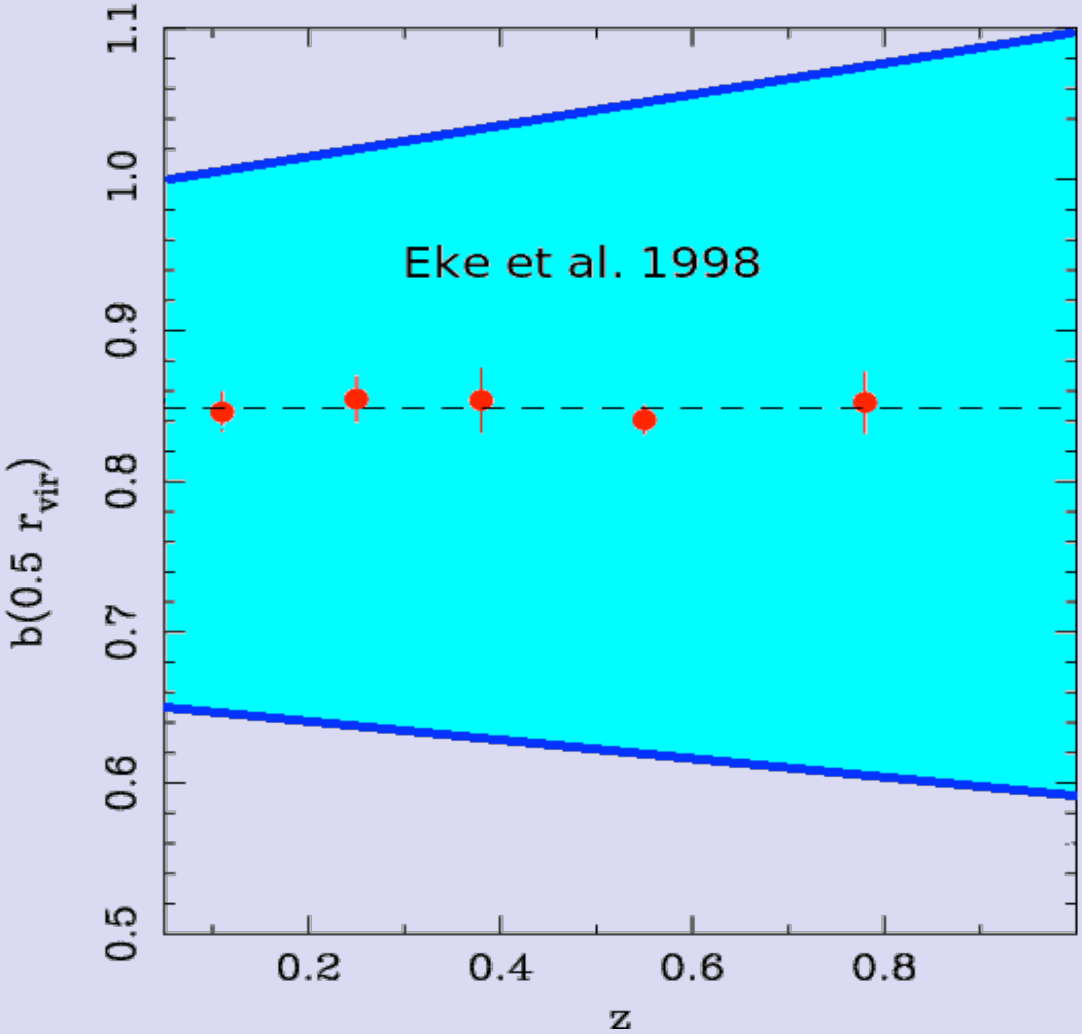
42 dynamically relax clusters

Hot kT>5keV

0.06<z<1.07

Scatter <~10% in f_{gas}

Allowances for systematic uncertainties



1) Gas depletion (simulation physics)

$b(z)=b_0(1+\alpha_b z)$

normalization:
20% uniform prior
$0.65 < b_0 < 1.0$

evolution:
10% at z=1 uniform prior
$-0.1 < \alpha_b < 0.1$

Allowances for systematic uncertainties

2) Instrument calibration and modelling (gas clumping, etc.)

1.0±0.1, 10% Gaussian prior on K

3) Baryonic mass in stars

$s(z)=s_0(1+\alpha_s z)$

normalization s_0: 30% Gaussian uncertainty (observational)

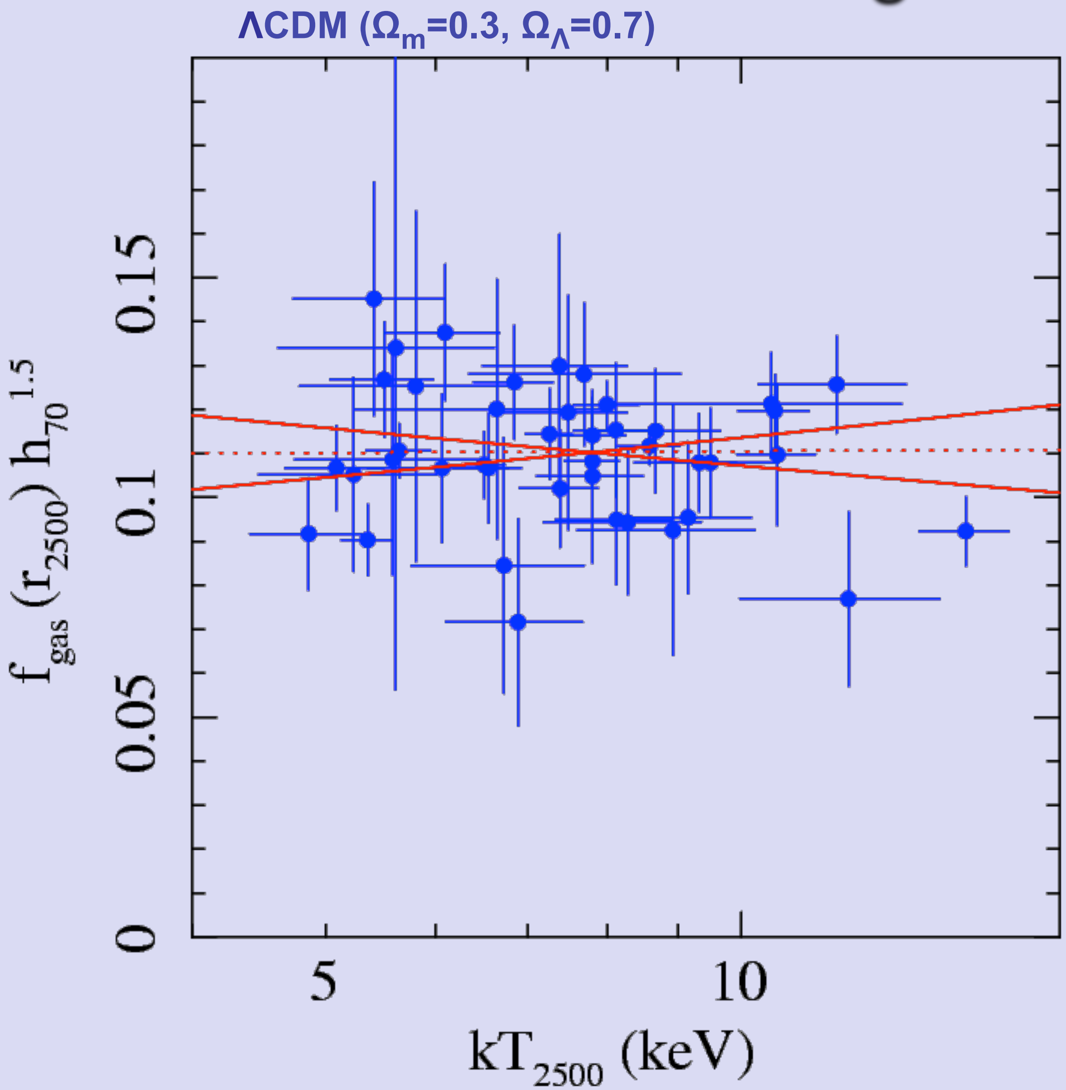
evolution $-0.2 < \alpha_s < 0.2$: 20% at z=1 uniform prior (observational)

4) Non-thermal pressure support in gas: (primarily due to bulk motions)

$\gamma = M_{true}/M_{X\text{-}ray}$

$1 < \gamma < 1.1$
10% uniform (Nagai et al 07, Werner et al 09, Sanders et al 09)

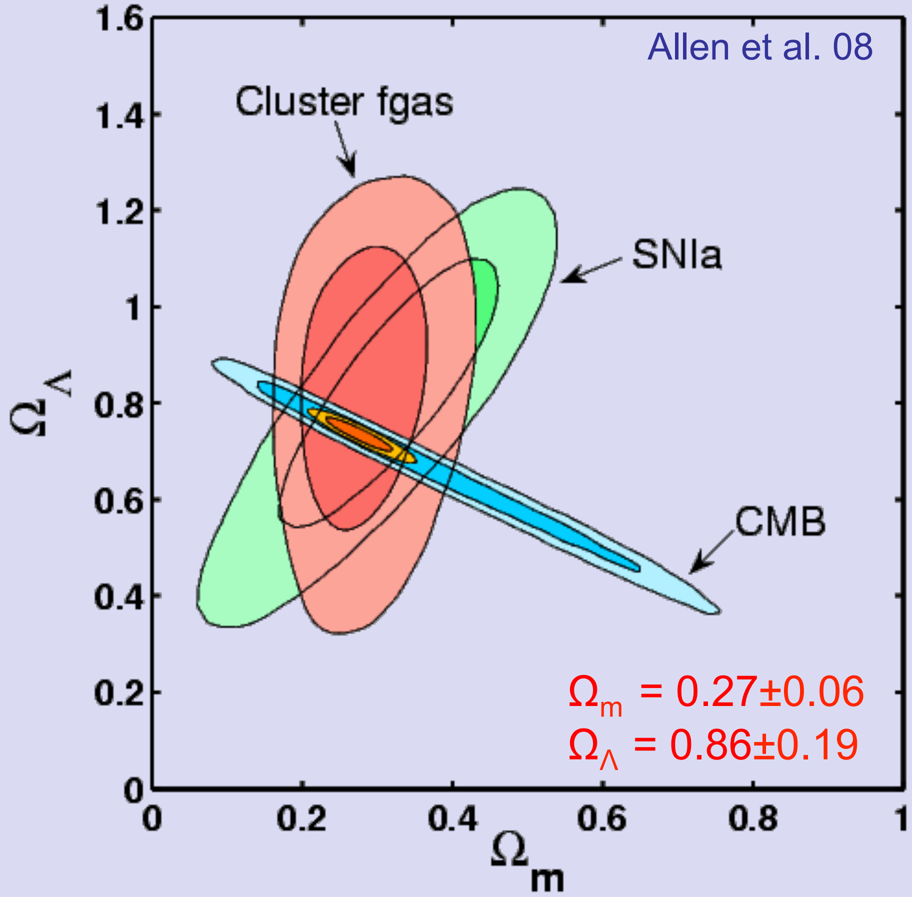
Chandra f_{gas}(kT) data



Best fitting power law: α=0.005±0.058 (solid lines 2σ limits).

f_{gas} essentially independent of temperature for the massive, dynamically relaxed clusters in the analysis.

Constraints on ΛCDM from three independent experiments



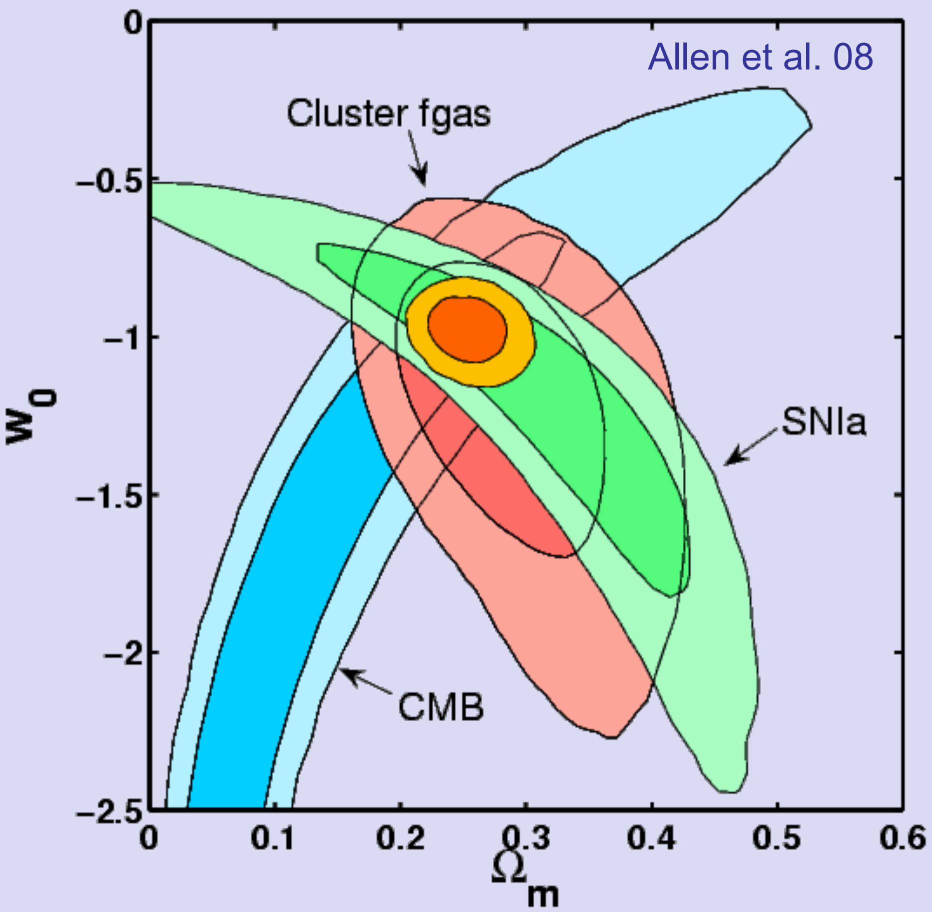
42 f_{gas} clusters (Allen et al 08) including standard BBNS+HST priors and full systematic allowances.

192 SNe Ia [Davis et al 07: Riess et al 07 (Gold sample), Wood-Vasey et al 07 (ESSENCE), Astier et al 06 (1rst year SNLS].

CMB data from WMAP3, CBI, Boomerang, ACBAR (prior 0.2<h<2.0).

Combined constraints
$\Omega_m = 0.275 \pm 0.033$
$\Omega_\Lambda = 0.735 \pm 0.023$

Constraints on wCDM from three independent experiments



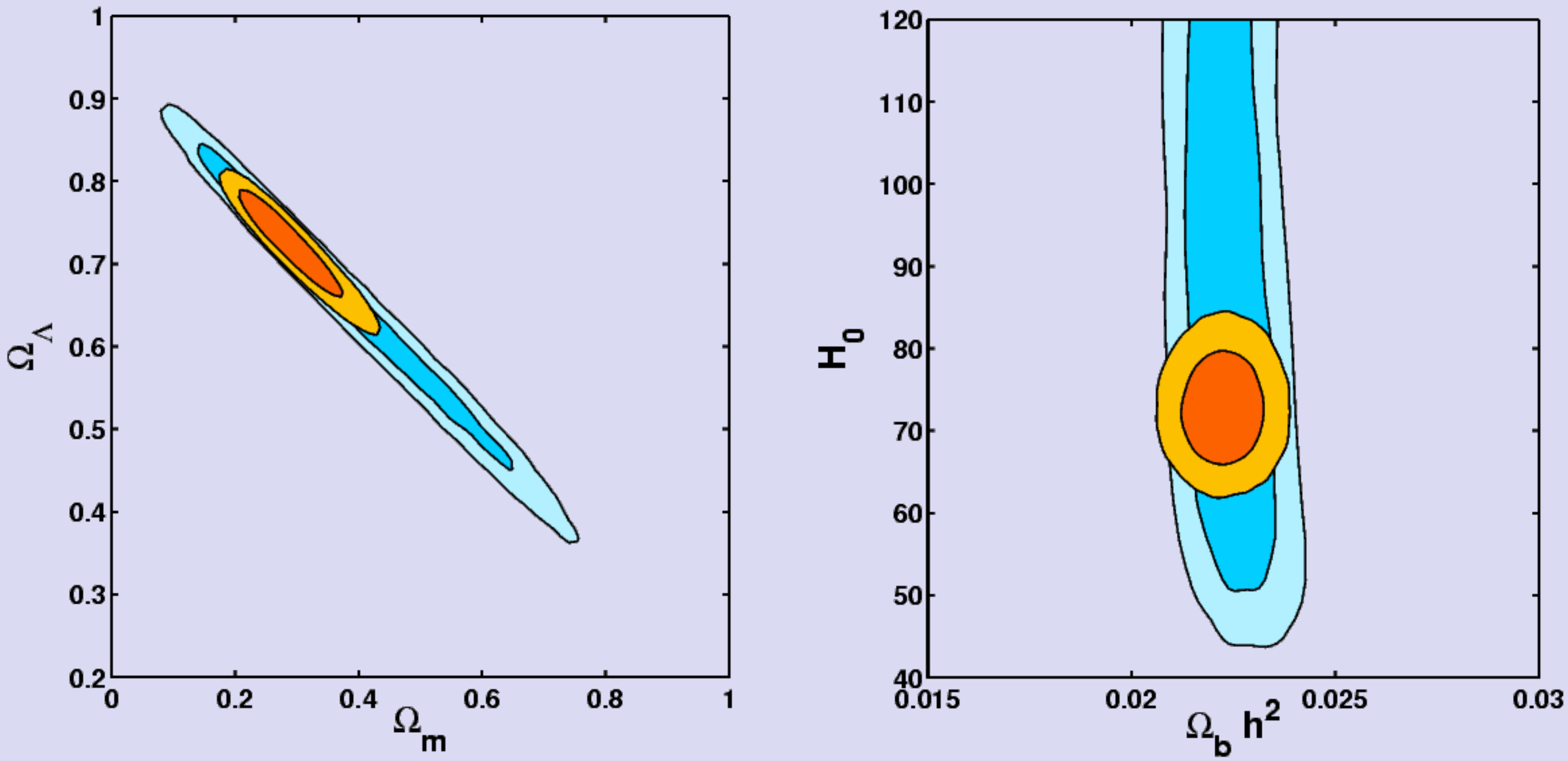
42 f_{gas} clusters (Allen et al 08) including standard BBNS+HST priors and full systematic allowances.

192 SNe Ia [Davis et al 07: Riess et al 07 (Gold sample), Wood-Vasey et al 07 (ESSENCE), Astier et al 06 (1rst year SNLS].

CMB data from WMAP3, CBI, Boomerang, ACBAR (prior 0.2<h<2.0).

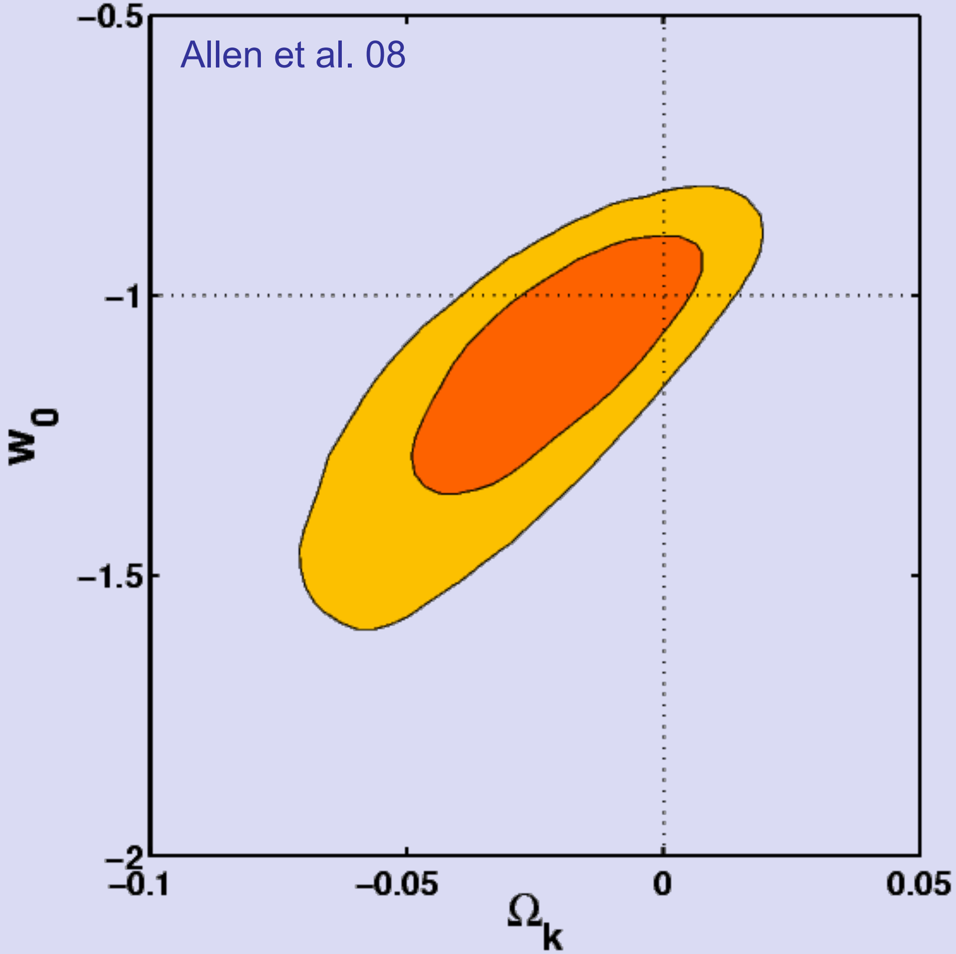
Combined constraints
$\Omega_m = 0.253 \pm 0.021$
$w_0 = -0.98 \pm 0.07$

Breaking degenarcy power



CMB constraints; CMB+f_{gas} constraints

Current constraints: non-flat constant w



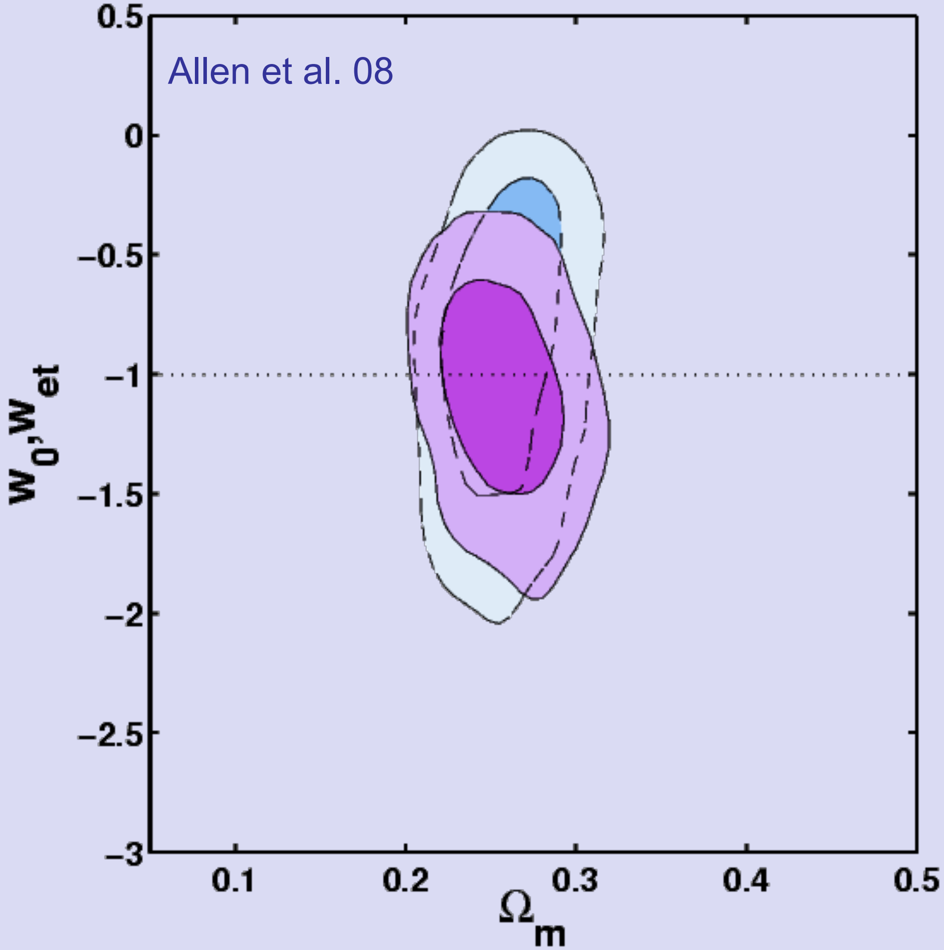
Orange: combined constraints.

Marginalized 68%

$\Omega_m = 0.312 \pm 0.052$

$w_0 = -1.08 + 0.13 - 0.19$

Current constraints: flat evolving w



Combined constraints

Marginalized 68%

$\Omega_m = 0.254 \pm 0.022$

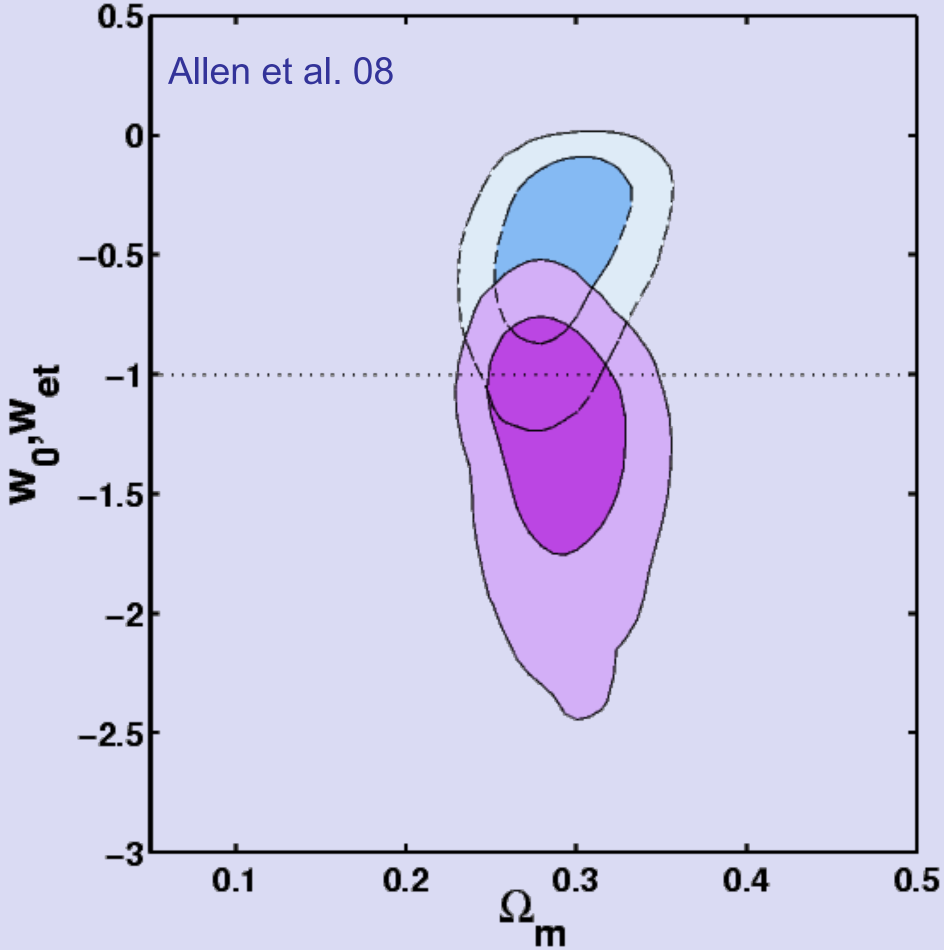
$w_0 = -1.05 + 0.31 - 0.26$

$w_{et} = -0.83 + 0.48 - 0.43$

marginalized over $0.05<z_t<1$

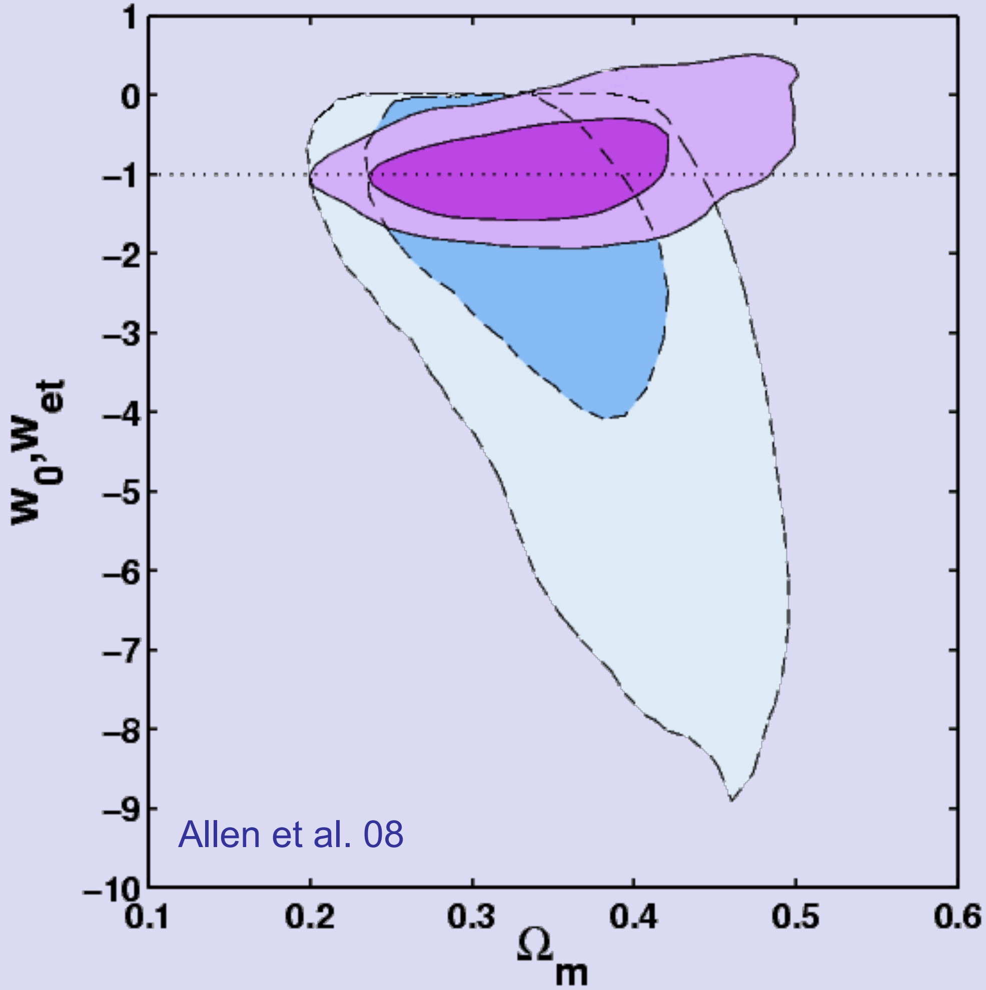
SNIa: Davis et al. 07

Current constraints: flat evolving w



Combined constraints
Marginalized 68%
$\Omega_m = 0.287 \pm 0.026$
$w_0 = -1.19 + 0.29 - 0.35$
$w_{et} = -0.33 + 0.18 - 0.34$
marginalized over $0.05 < z_t < 1$
SNIa: Riess et al. 07

Current constraints: non-flat evolving w

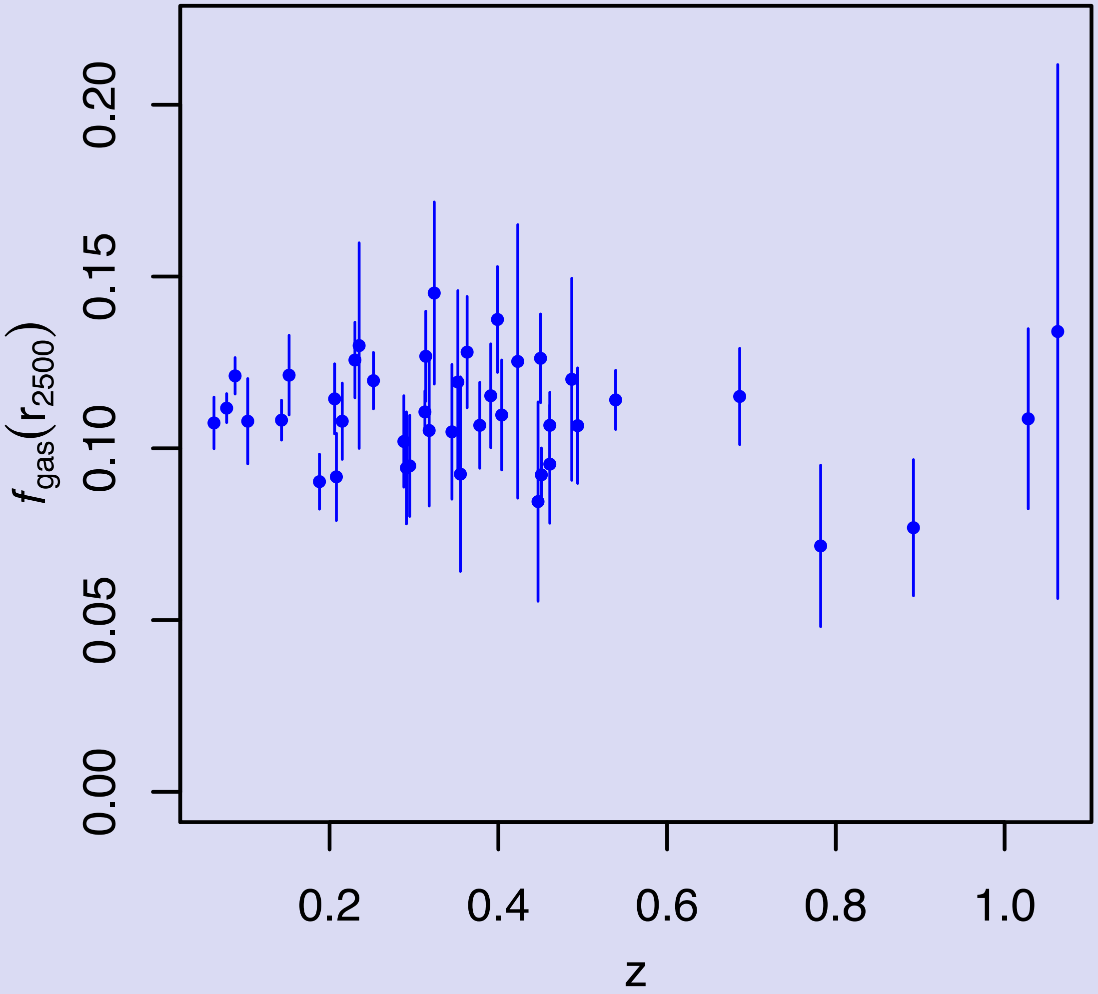


Combined constraints
Marginalized 68%
$\Omega_m = 0.29 + 0.09 - 0.04$
$w_0 = -1.15 + 0.50 - 0.38$
$w_{et} = -0.80 + 0.70 - 1.30$
marginalized over $0.05<z_t<1$

Preliminary new f_{gas} data

Allen et al 2012 (in prep)

Gas mass fraction: data
Low scatter total mass proxy



ΛCDM (Ω_m=0.3, Ω_Λ=0.7)

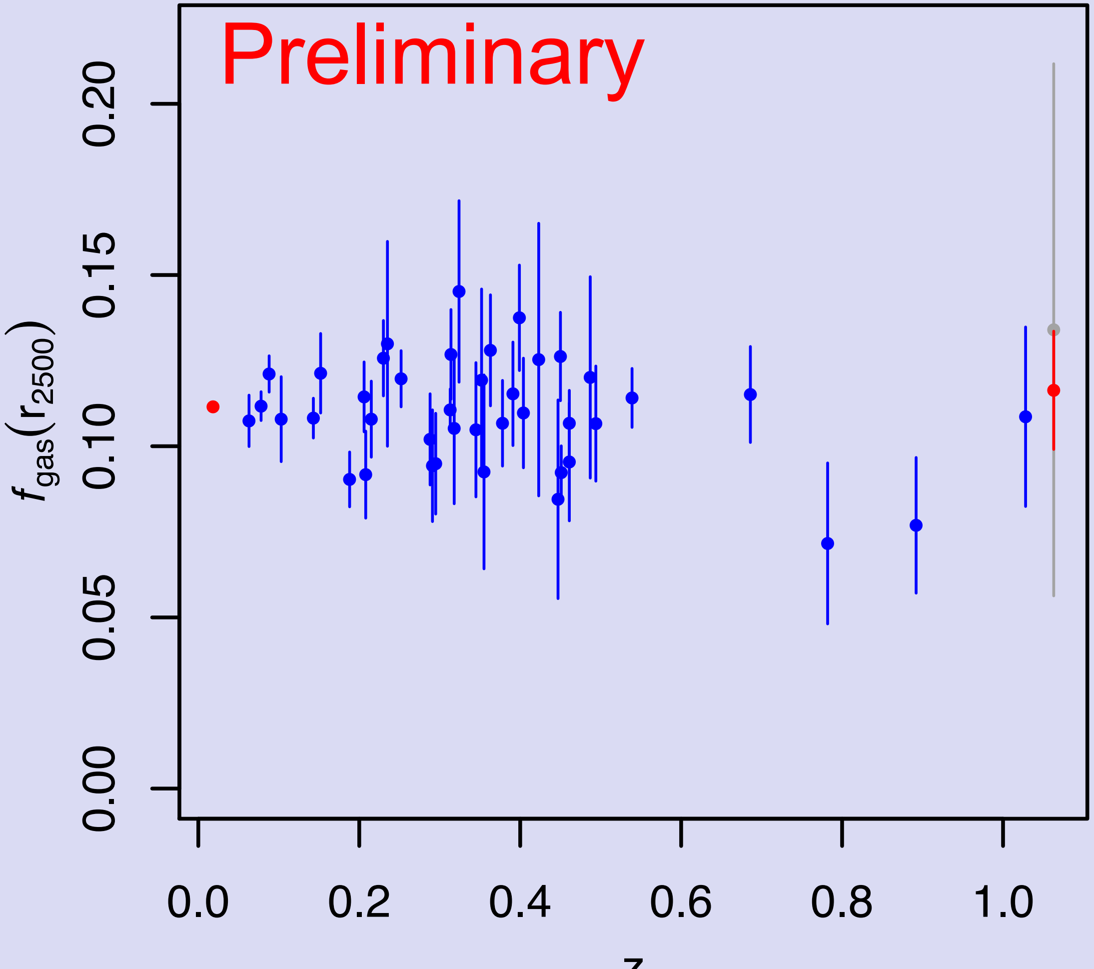
42 dynamically relax clusters

Hot kT>5keV

0.06<z<1.07

Scatter <~10% in f_{gas}

Gas mass fraction: data
Low scatter total mass proxy



ΛCDM (Ω_m=0.3, Ω_Λ=0.7)

42 dynamically relax clusters

Hot kT>5keV

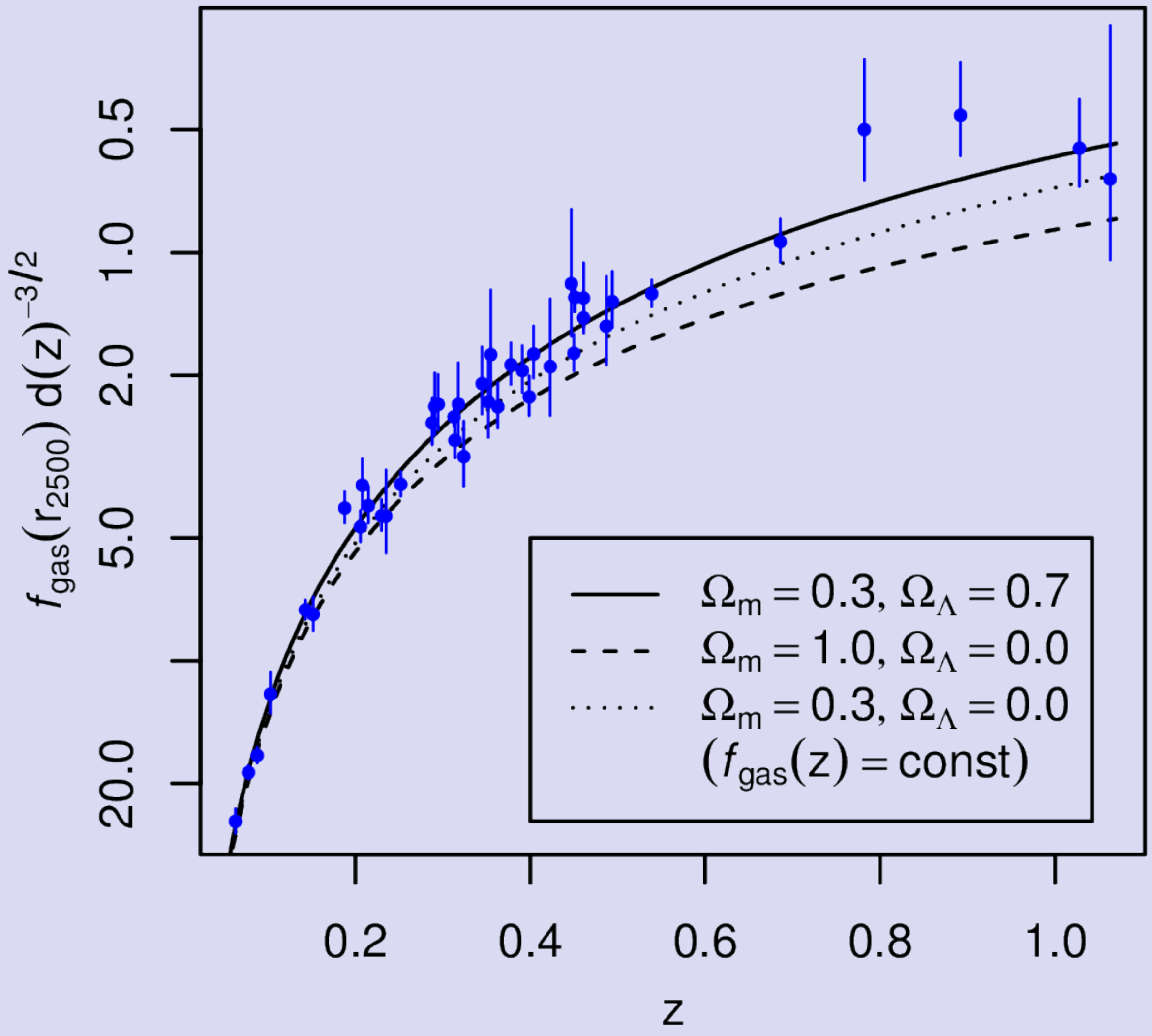
0.06<z<1.07

Preliminary: New data

Perseus (low-z of 0.018)

3c186 (high-z of 1.06)

Measured quantity: $f_{gas}(r_{2500})d(z)^{-3/2}$



ΛCDM (Ω_m=0.3, Ω_Λ=0.7)

42 dynamically relax clusters

Hot kT>5keV

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Scatter <~10%

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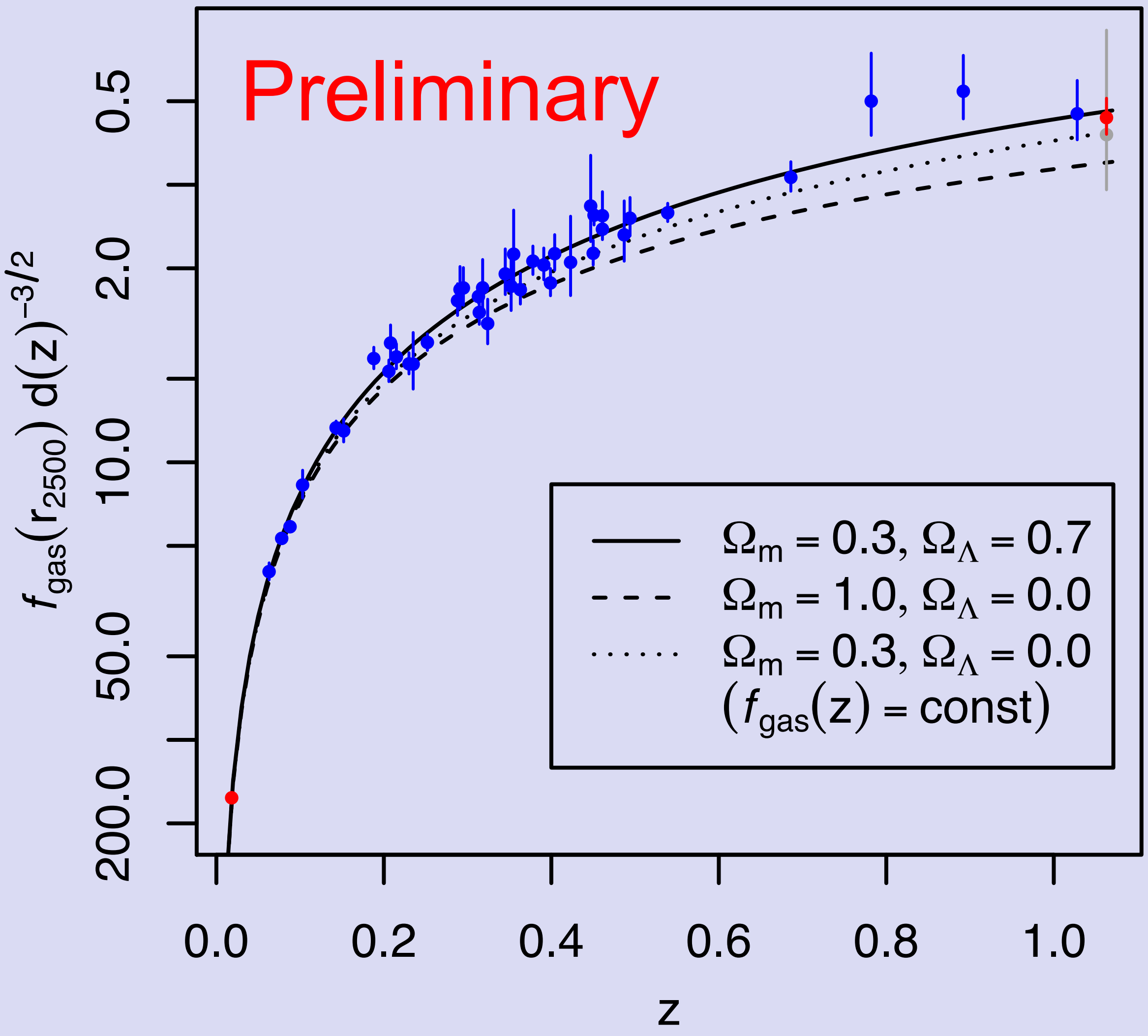

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42 dynamically relax clusters

Hot kT>5keV

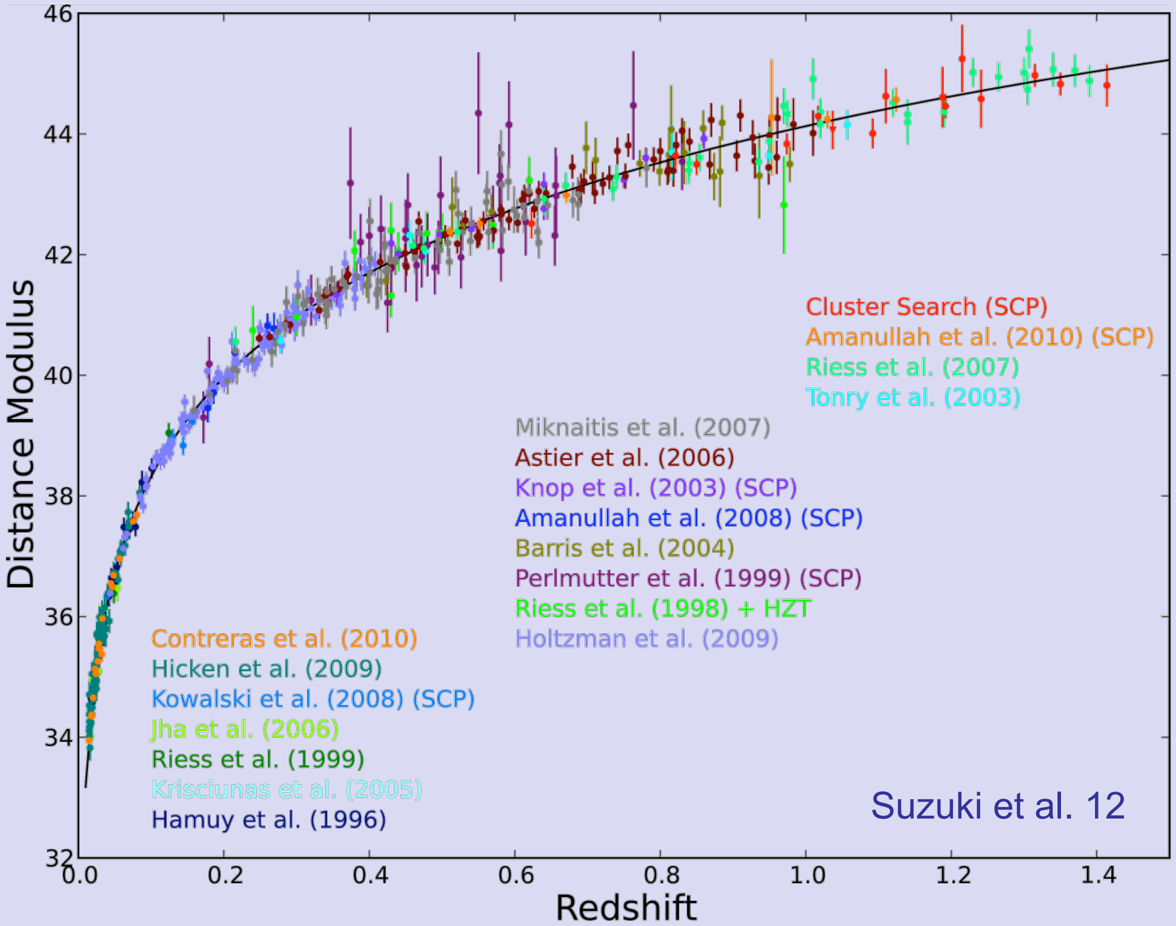
0.06<z<1.07

Preliminary: New data

Perseus (low-z of 0.018)

3c186 (high-z of 1.06)

Measured quantity: magnitude: prop $d(z)^{-2}$



Supernovae data:
Union 2.1

580 Type Ia supernovae

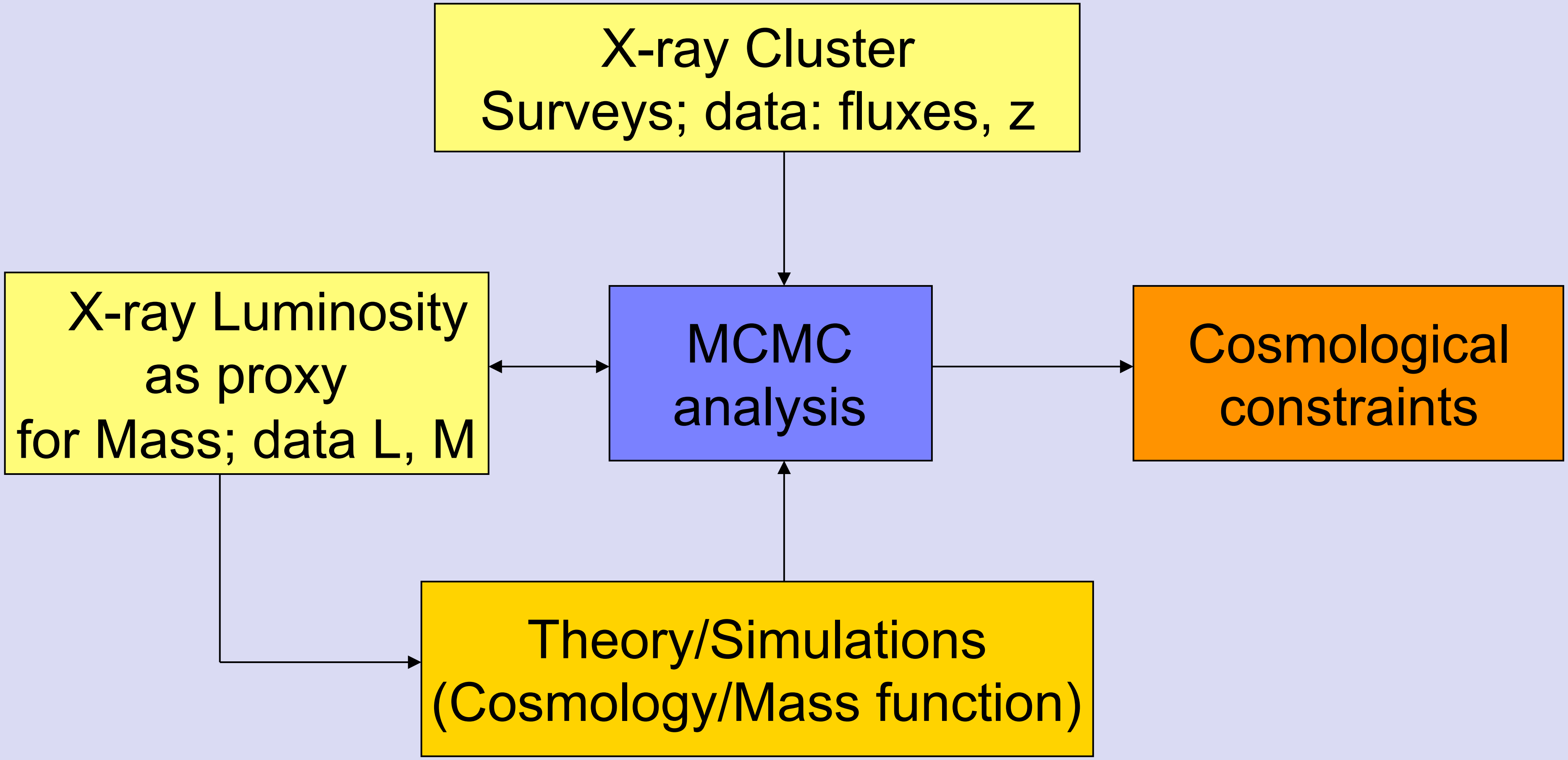
$z<1.415$

Cluster abundance and scaling relations

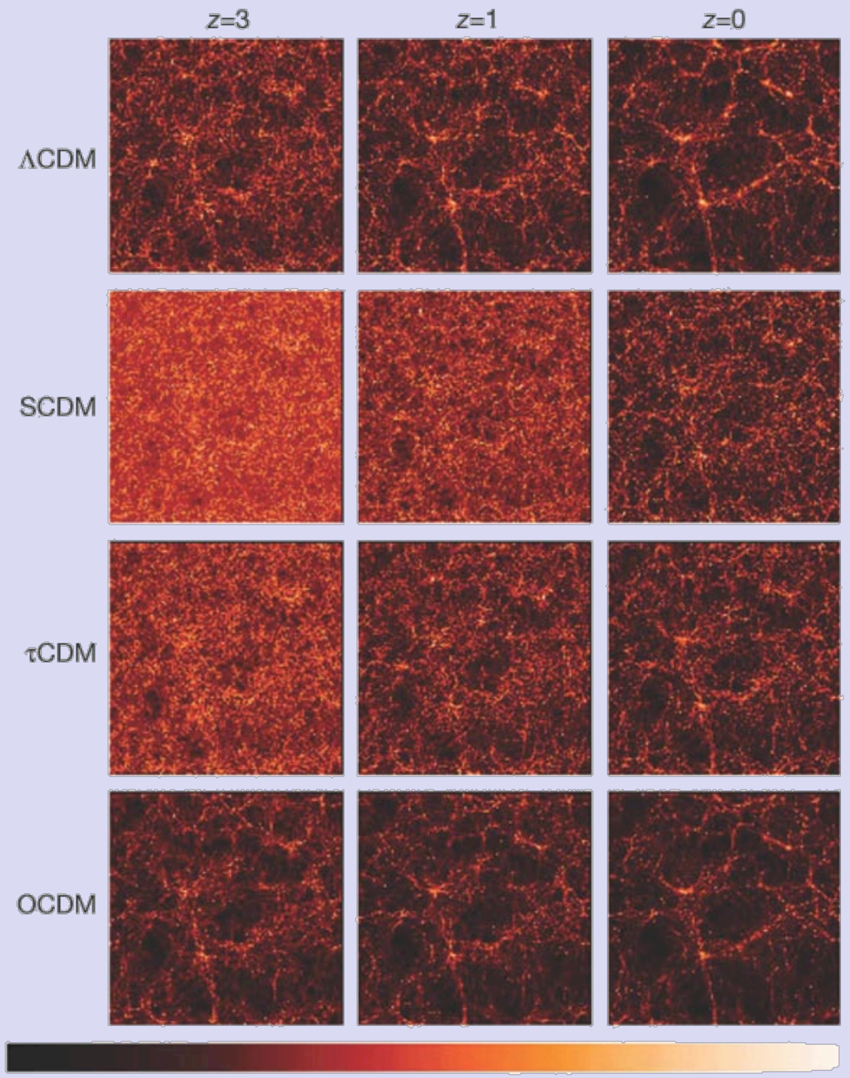
e.g. Mantz et al 08, 10a, 10b; Vikhlinin et al 09;
Rapetti et al. 09, 10; Schmidt et al 09

Basic initial idea:

Light yellow: Data
Dark yellow: Model
Blue: Analysis
Orange: Product



Theory: Growth of structure

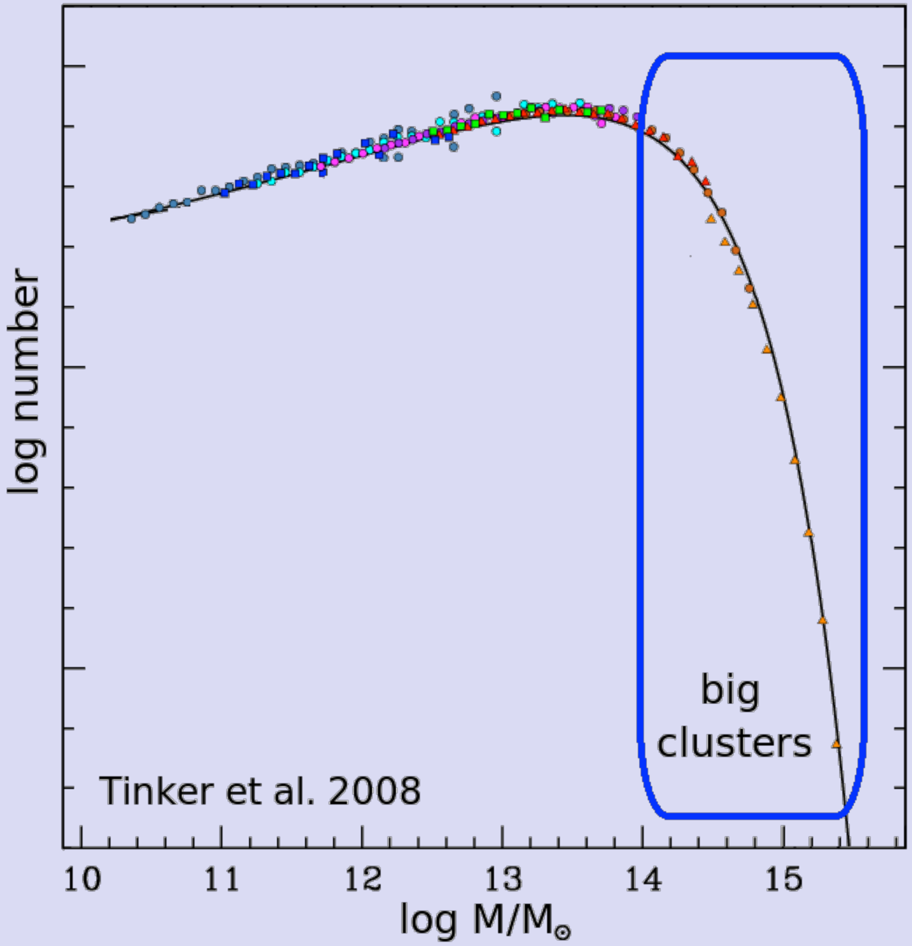


- Simulated cosmologies to model the non-linear growth of structure.

- Even looking so apparently different can be conveniently related with the linear growth calculations through a fitting formula. (See e.g. Jenkins et al 2001, Tinker et al 2008, etc.)

Cole et al 2005

Cluster abundance as a function of mass and redshift

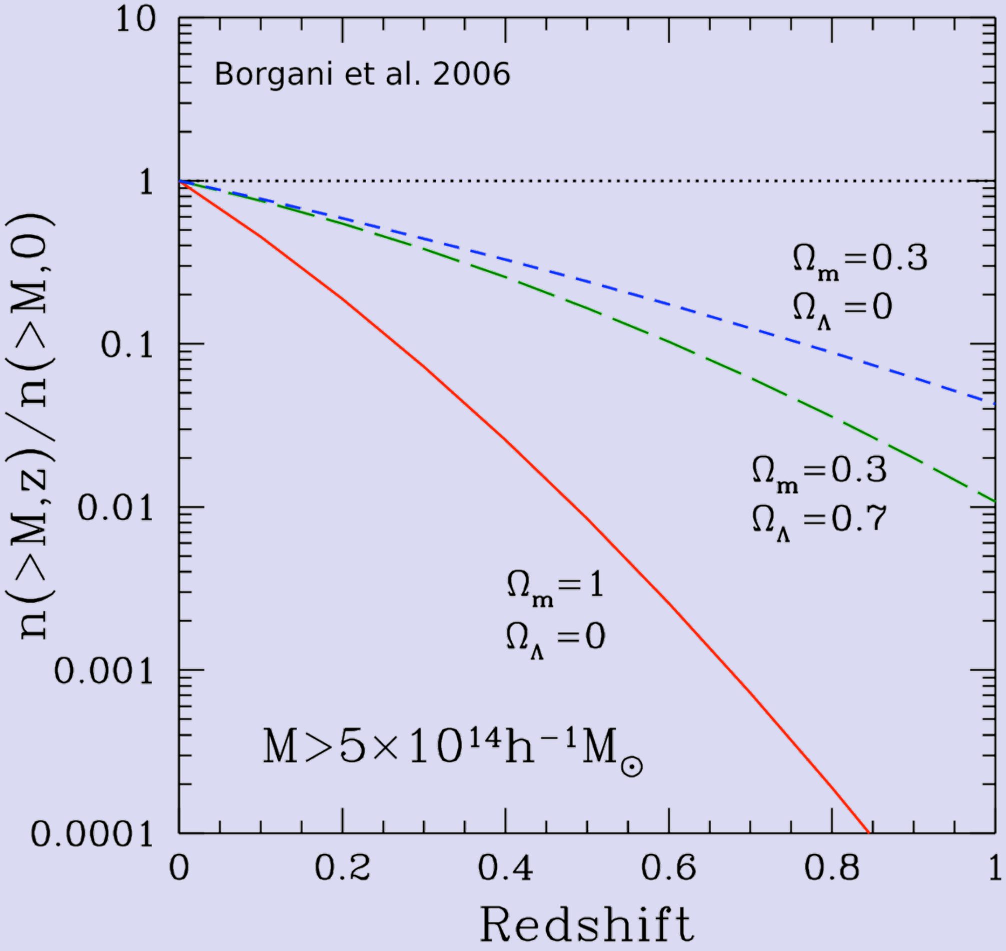


N-body simulations

Non-linear structure formation

Big clusters steep mass function; sensitive to the cosmological model; quintessence, self-interacting, early, clustering dark energy as well as modified gravity

Cluster abundance as a function of mass and redshift



Linear theory

Sensitive to the cosmological model; quintessence, self-interacting, early, clustering dark energy as well as modified gravity

Modern cosmology with X-ray luminous clusters of galaxies

Tuesday Lecture/Practice: cosmological codes and MCMC techniques

David Rapetti

DARK Fellow

Dark Cosmology Centre, Niels Bohr Institute

Dark Cosmology Centre

University of Copenhagen

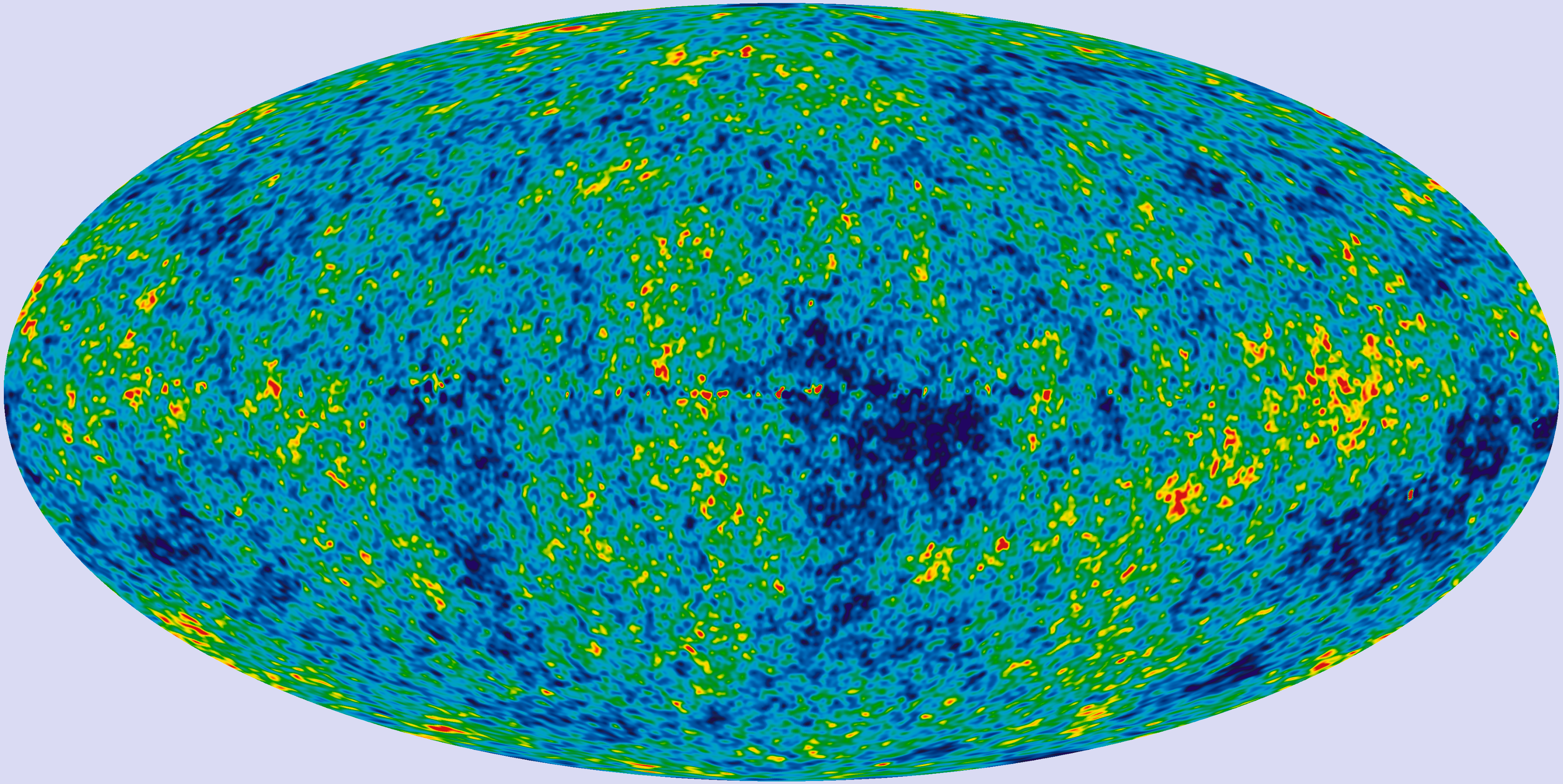
Install CosmoMC and CAMB

- After having CosmoMC and CAMB downloaded from http://cosmologist.info/cosmomc/ and the fgas module from http://www.slac.stanford.edu/~drapetti/fgas_module/, follow the installation instructions in the corresponding websites: do not hesitate to ask questions in class whenever needed.

- Go into the CAMB folder in CosmoMC and compile it using an appropriate Fortran compiler according with your choices in your makefile.

- Later on (after the CAMB exercises) repeat the same operation to compile CosmoMC working within the source folder.

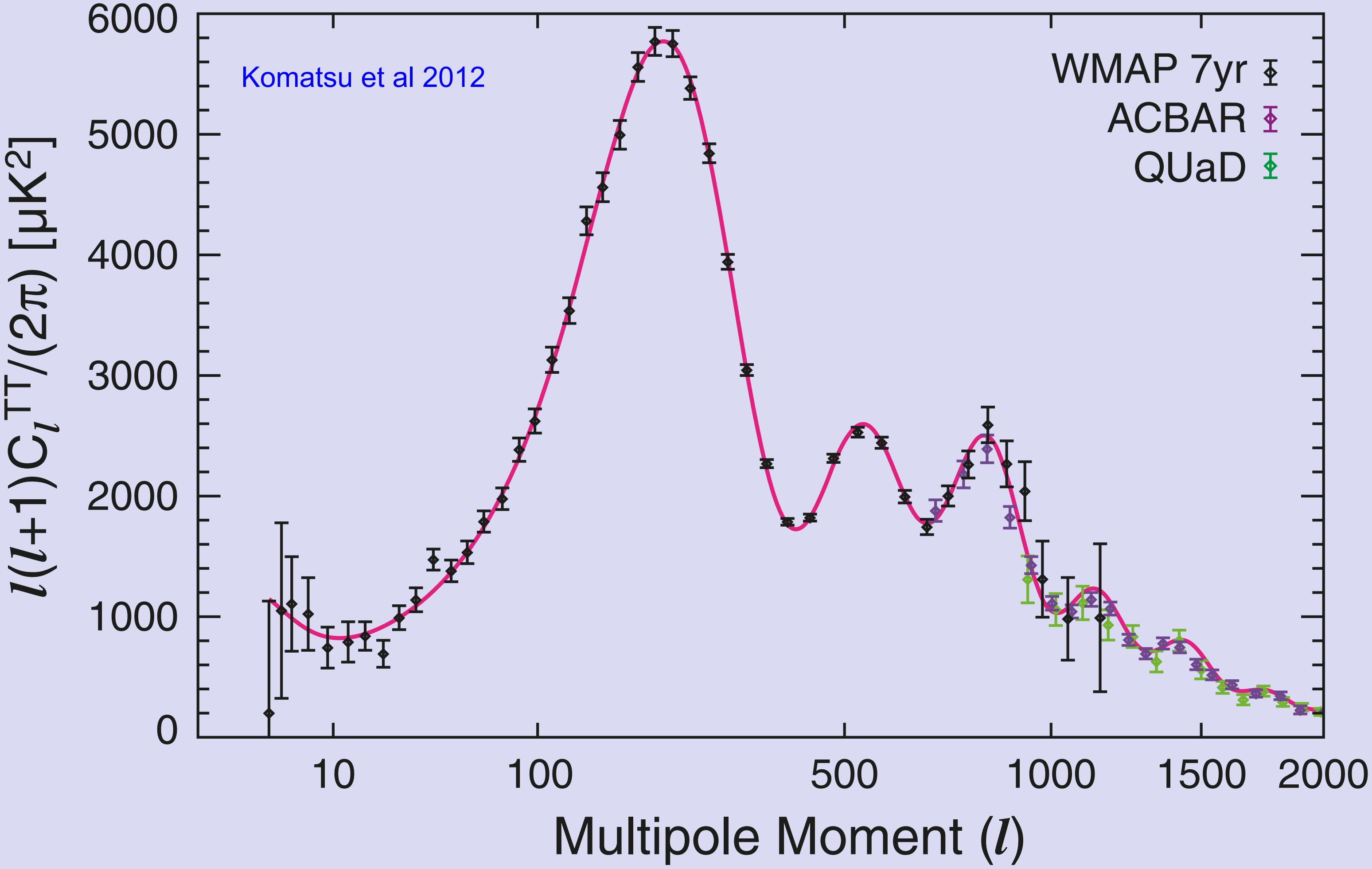
Current measurements from NASA's WMAP satellite



To calculate the power spectrum we can use CAMB (see e.g. the connection between such a map and the power spectrum in this link: http://background.uchicago.edu/~whu/metaanim.html)

Cosmic Microwave Background (CMB)

Wilkinson Microwave Anisotropy Map (WMAP): currently, 7 years results (together with ACBAR and QUaD data)



Practice/exercises with CAMB

CMB exercises using CAMB

- Using the WMAP7 cosmology calculate the theoretical curve of the previous figure. Plot the results (use your favorite plotting program).

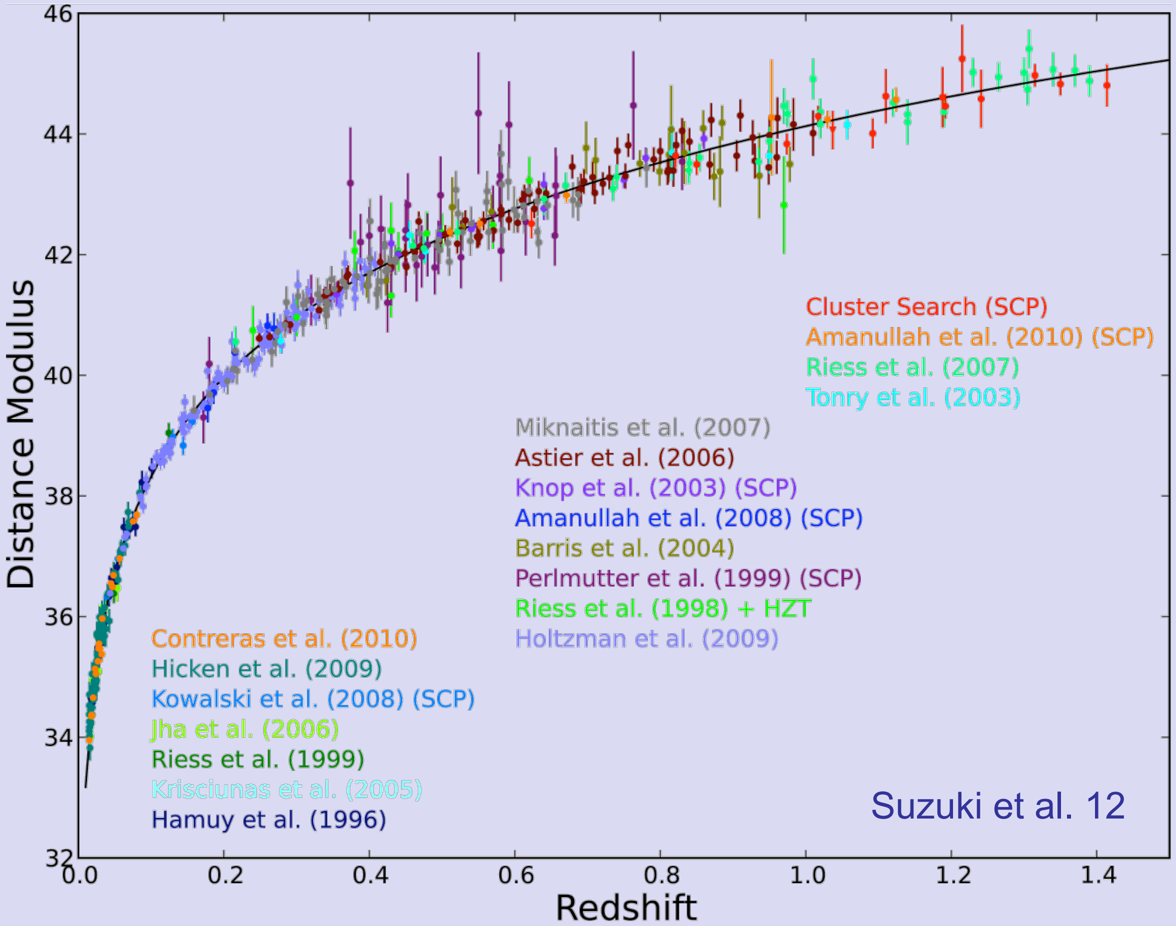
- Change one cosmological parameter at a time to explore how it modifies the curve.

- Repeat the above operation for various cosmological parameters (you can check the following website for inspiration and for comparison: http://background.uchicago.edu/~whu/)

- Remember to ask when needed to be able to move on to the next exercises timely.

Practice/exercises with CosmoMC

Type Ia supernovae



Supernovae data:
Union 2.1

580 Type Ia supernovae

z<1.415

Type Ia supernovae

Distance modulus

$$\mu(z) \equiv m - M = 5\log_{10}(d_L/10\,\mathrm{pc}) = 5\log_{10}[(1+z)r(z)/\mathrm{pc}] - 5$$

From Suzuki et al 2012, ApJ, 746, 85

$$\mu_B = m_B^{\max} + \alpha \cdot x_1 - \beta \cdot c + \delta \cdot P\left(m_\star^{\mathrm{true}} < m_\star^{\mathrm{threshold}}\right) - M_B$$

and the fitting procedure:

$$\chi^2_{\mathrm{stat}} = \sum_{\mathrm{SNe}} \frac{[\mu_B(\alpha, \beta, \delta, M_{\mathrm{B}}) - \mu(z; \Omega_m, \Omega_w, w)]^2}{\sigma^2_{\mathrm{lc}} + \sigma^2_{\mathrm{ext}} + \sigma^2_{\mathrm{sample}}}$$

Exercise: Use the SNe Ia code of the Union 2.1 in CosmoMC to obtain the constraints on the paper (Suzuki et al 12). You can also use the SCP website http://supernova.lbl.gov/Union/.

X-ray gas mass fraction experiment

$$f_{gas}^{ref}(z) = \frac{b(z)\gamma K}{1+s(z)}\left(\frac{\Omega_b}{\Omega_m}\right)\varepsilon(\theta)\left[\frac{d_A^{ref}(z)}{d_A^{\mathrm{mod}}(z;\theta)}\right]^{3/2}$$

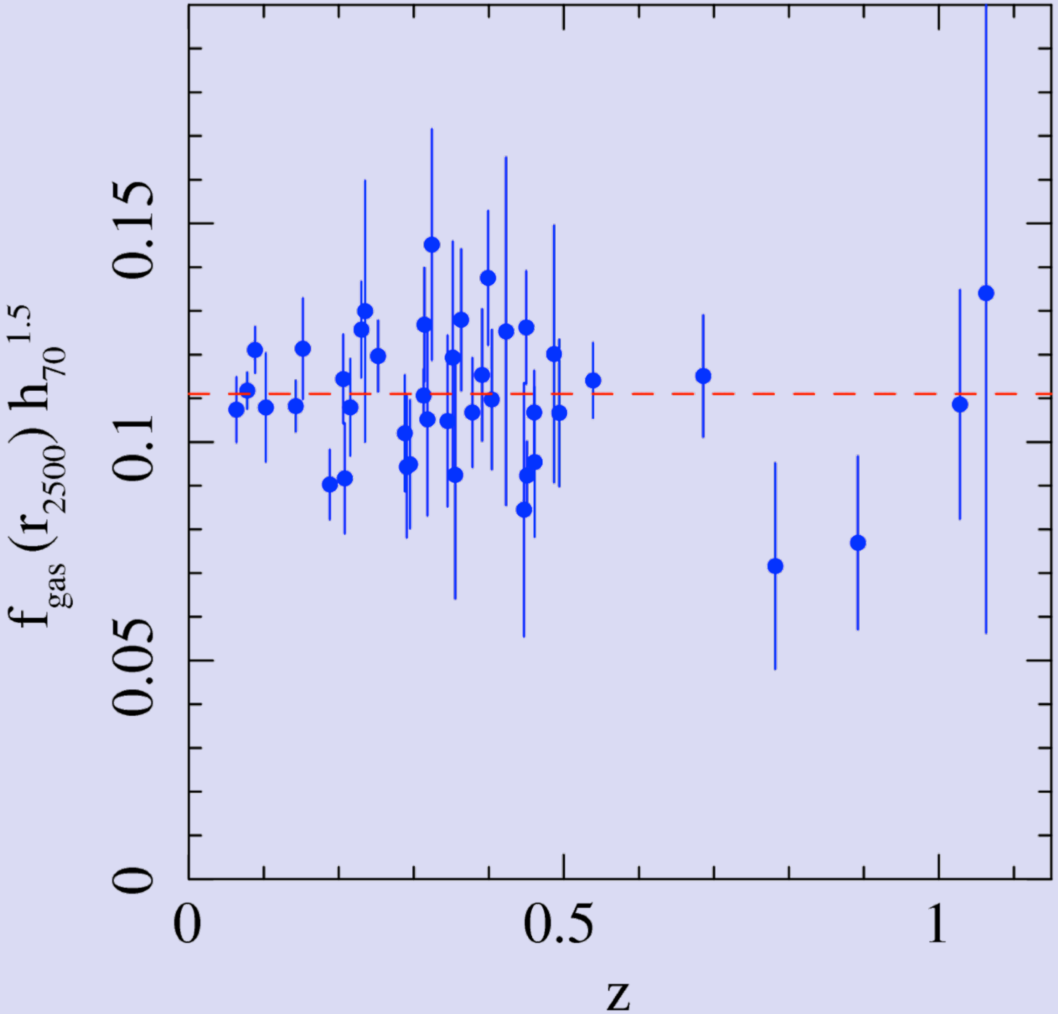
Apparent evolution of the gas mass fraction

$$\varepsilon(\theta) = \left[\frac{H^{\mathrm{mod}}(z;\theta)\,d_A^{\mathrm{mod}}(z;\theta)}{H^{ref}(z)\,d_A^{ref}(z)}\right]^{\eta}$$

Small angular correction

$\eta = 0.214 \pm 0.022$ Measured from the data profiles

X-ray gas mass fraction data



ΛCDM (Ω_m=0.3, Ω_Λ=0.7)

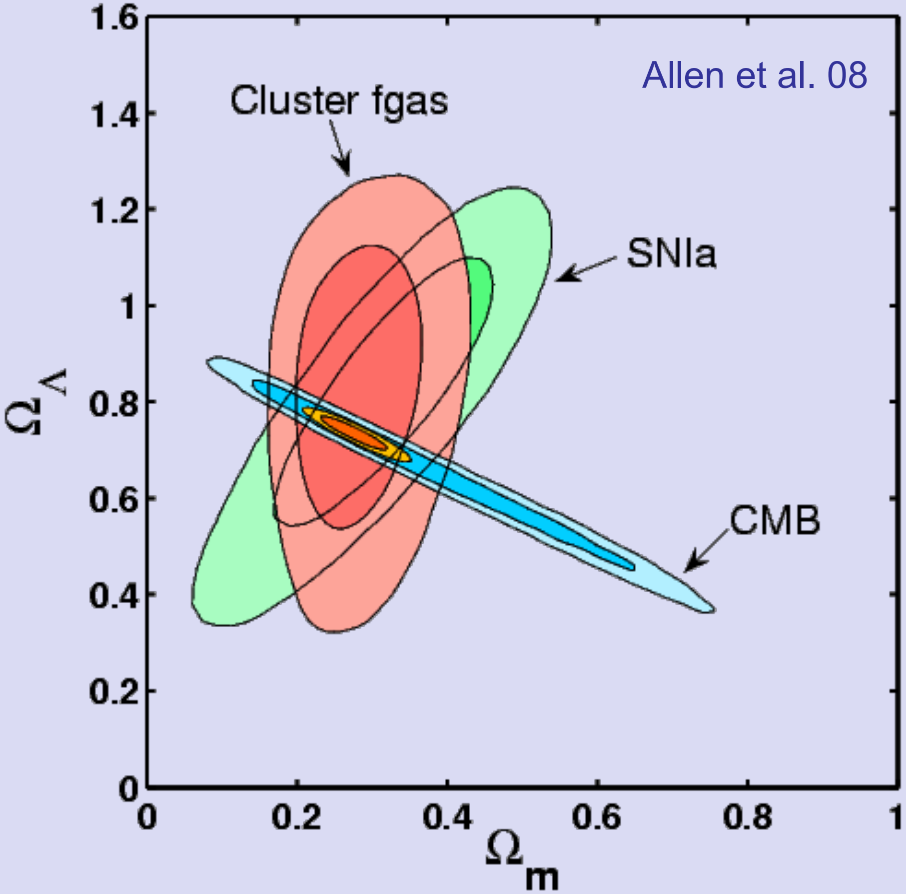
42 dynamically relax clusters

Hot kT>5keV

0.06<z<1.07

Exercise: Plot the data from the data folder in cosmomc (with the corresponding error bars) to get familiar with it.

Constraints on ΛCDM

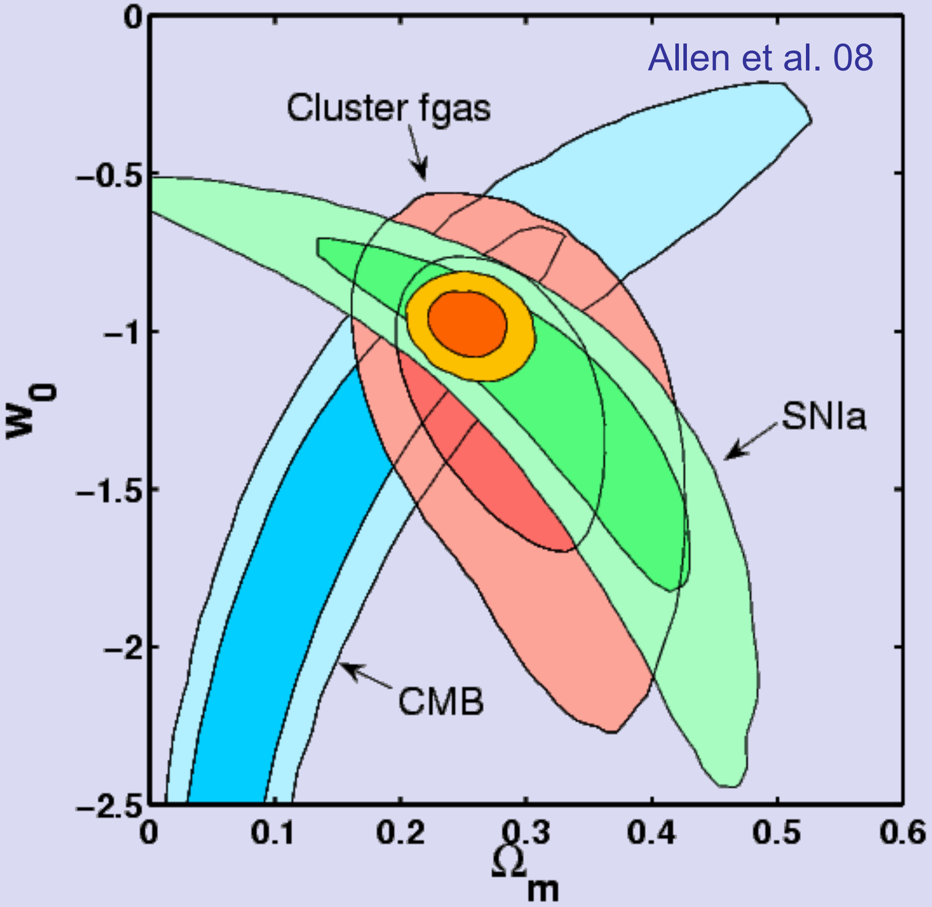


42 f_{gas} clusters (Allen et al 08) including standard BBNS+HST priors and full systematic allowances.

Exercise: Reproduce the clusters, red constraints using the fgas module in CosmoMC; obtain also the marginalized constraints below.

$\Omega_m = 0.27 \pm 0.06$
$\Omega_\Lambda = 0.86 \pm 0.19$

Constraints on wCDM



42 f_{gas} clusters (Allen et al 08) including standard BBNS+HST priors and full systematic allowances.

Exercise: Reproduce the clusters, red constraints using the fgas module in CosmoMC.

Systematic uncertainty parameters

1) Gas depletion (simulation physics)

$b(z)=b_0(1+\alpha_b z)$
normalization: 20% uniform prior $0.65 < b_0 < 1.0$
evolution: 10% at z=1 uniform prior $-0.1 < \alpha_b < 0.1$

2) Instrument calibration and modelling (gas clumping, etc.)

1.0±0.1, 10% Gaussian prior on K

Exercise: Test their robustness by sensibly changing the allowances in the fgas module in CosmoMC and obtaining new constraints

3) Baryonic mass in stars

$s(z)=s_0(1+\alpha_s z)$
normalization s_0: 30% Gaussian uncertainty (observational)
evolution $-0.2 < \alpha_s < 0.2$: 20% at z=1 uniform prior (observational)

4) Non-thermal pressure support in gas: (primarily due to bulk motions)

$\gamma = M_{true}/M_{X\text{-}ray}$
$1< \gamma <1.1$ 10% uniform (simulations/observations)

Modern cosmology with X-ray luminous clusters of galaxies

Wednesday Lecture: Cluster Abundance Cosmology

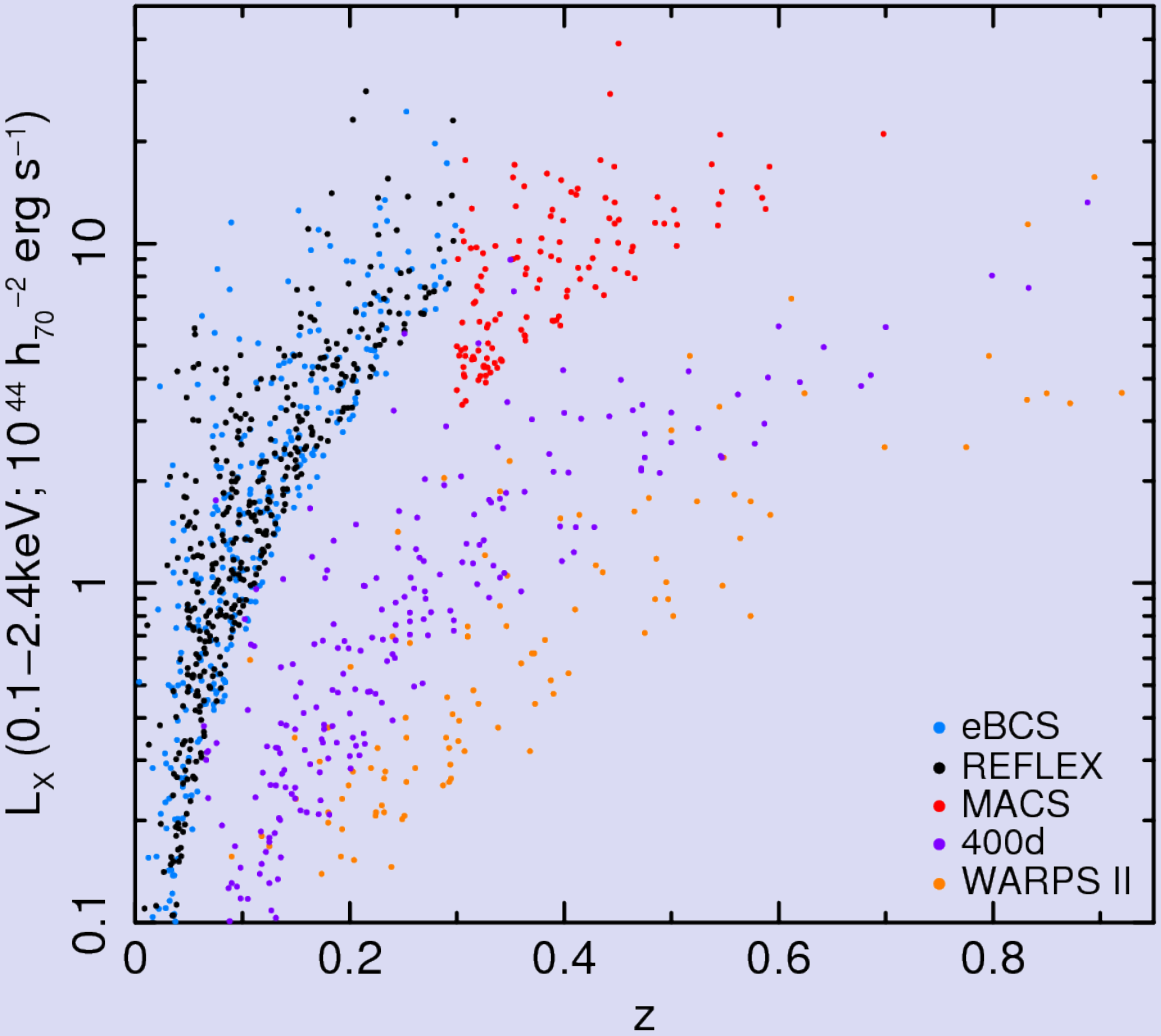
David Rapetti

DARK Fellow

Dark Cosmology Centre, Niels Bohr Institute

Dark Cosmology Centre University of Copenhagen

Cluster surveys



Full cosmological analysis in this series of papers

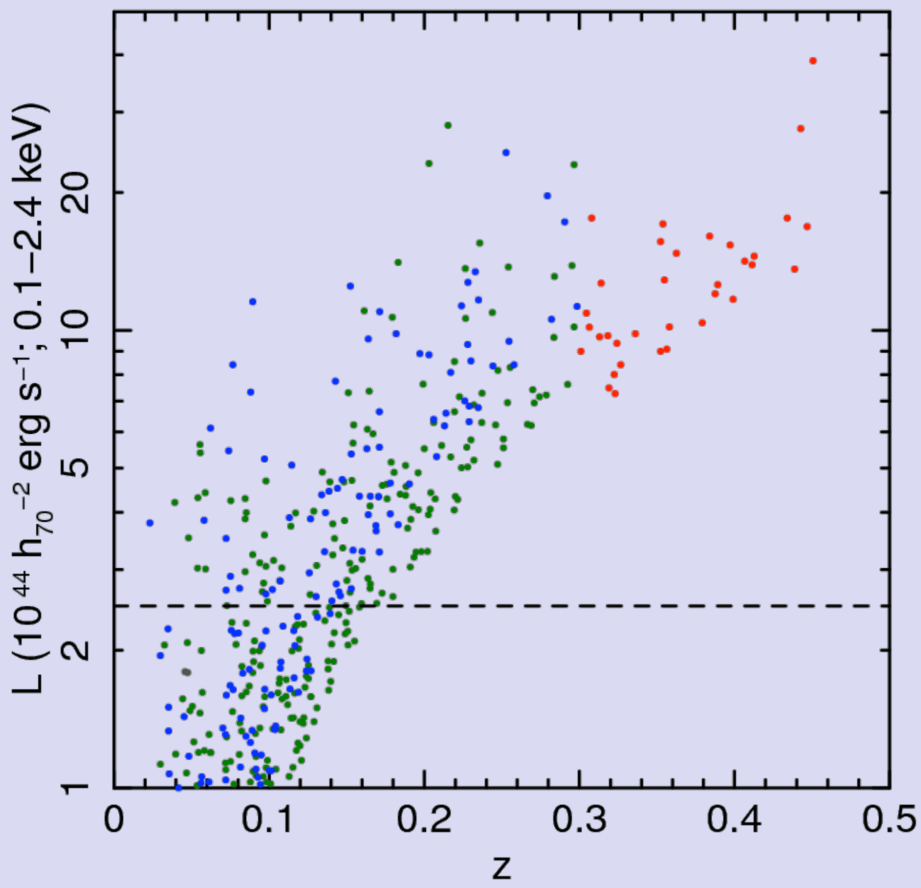
"The Observed Growth of Massive Galaxy Clusters I: Statistical Methods and Cosmological Constraints",
MNRAS 406, 1759, 2010
Adam Mantz, Steven Allen, David Rapetti, Harald Ebeling

"The Observed Growth of Massive Galaxy Clusters II: X-ray Scaling Relations",
MNRAS 406, 1773, 2010
Adam Mantz, Steven Allen, Harald Ebeling, David Rapetti, Alex Drlica-Wagner

"The Observed Growth of Massive Galaxy Clusters III: Testing General Relativity at Cosmological Scales",
MNRAS 406, 1796, 2010
David Rapetti, Steven Allen, Adam Mantz, Harald Ebeling
(Chandra/NASA press release together with Schmidt, Vikhlinin & Hu 09,
April 14 2010, "Einstein's Theory Fights off Challengers")

"The Observed Growth of Massive Galaxy Clusters IV: Robust Constraints on Neutrino Properties",
MNRAS 406, 1805, 2010
Adam Mantz, Steven Allen, David Rapetti

Cluster survey data



L > 2.55×10^{44} h_{70}^{-2} erg s^{-1} (dashed line).
Cuts leave 78+126+34=238 massive clusters

Low redshift (z<0.3)

- BCS (Ebeling et al 98, 00)
  F > 4.4×10^{-12} erg s^{-1} cm^{-2}
  ~33% sky coverage
- REFLEX (Böhringer et al 04)
  F > 3.0×10^{-12} erg s^{-1} cm^{-2}
  ~33% sky coverage

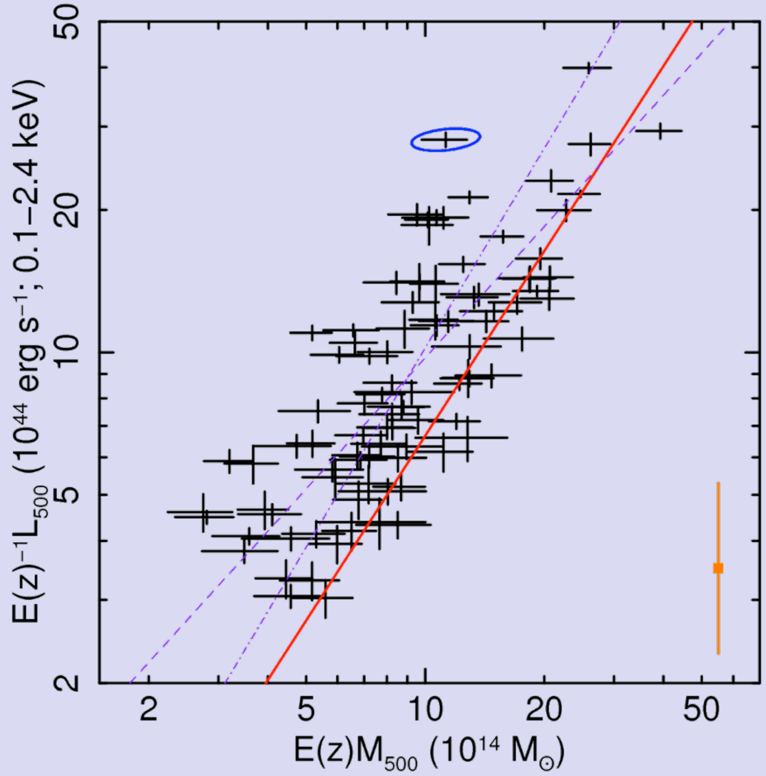
Intermediate redshifts (0.3<z<0.5)

- Bright MACS (Ebeling et al 01, 10)
  F > 2.0×10^{-12} erg s^{-1} cm^{-2}
  ~55% sky coverage

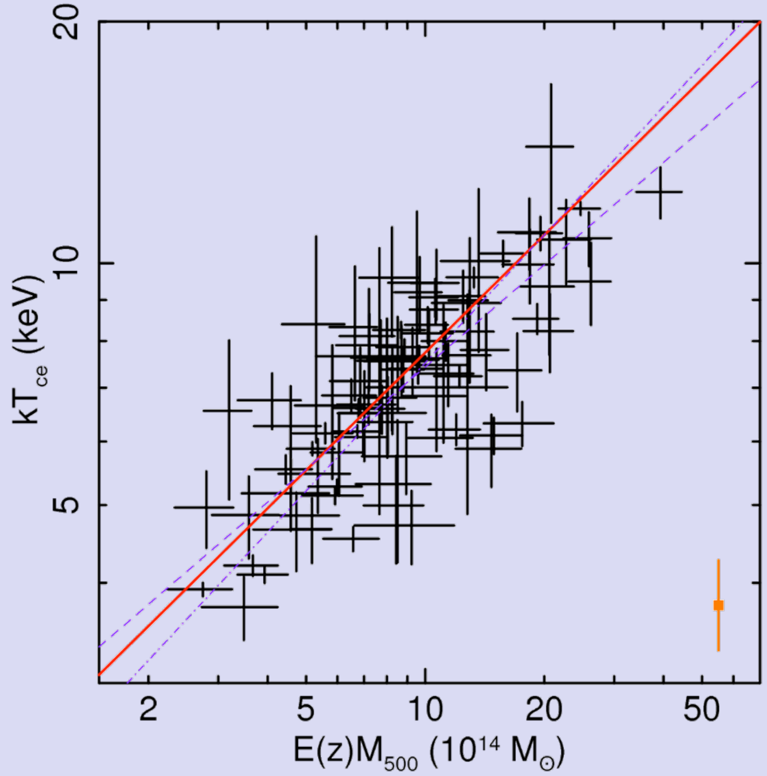
All based on RASS detections. Continuous and all 100% redshift complete.

Scaling relations data: X-ray follow-up for 94 clusters

Mantz et al 10b



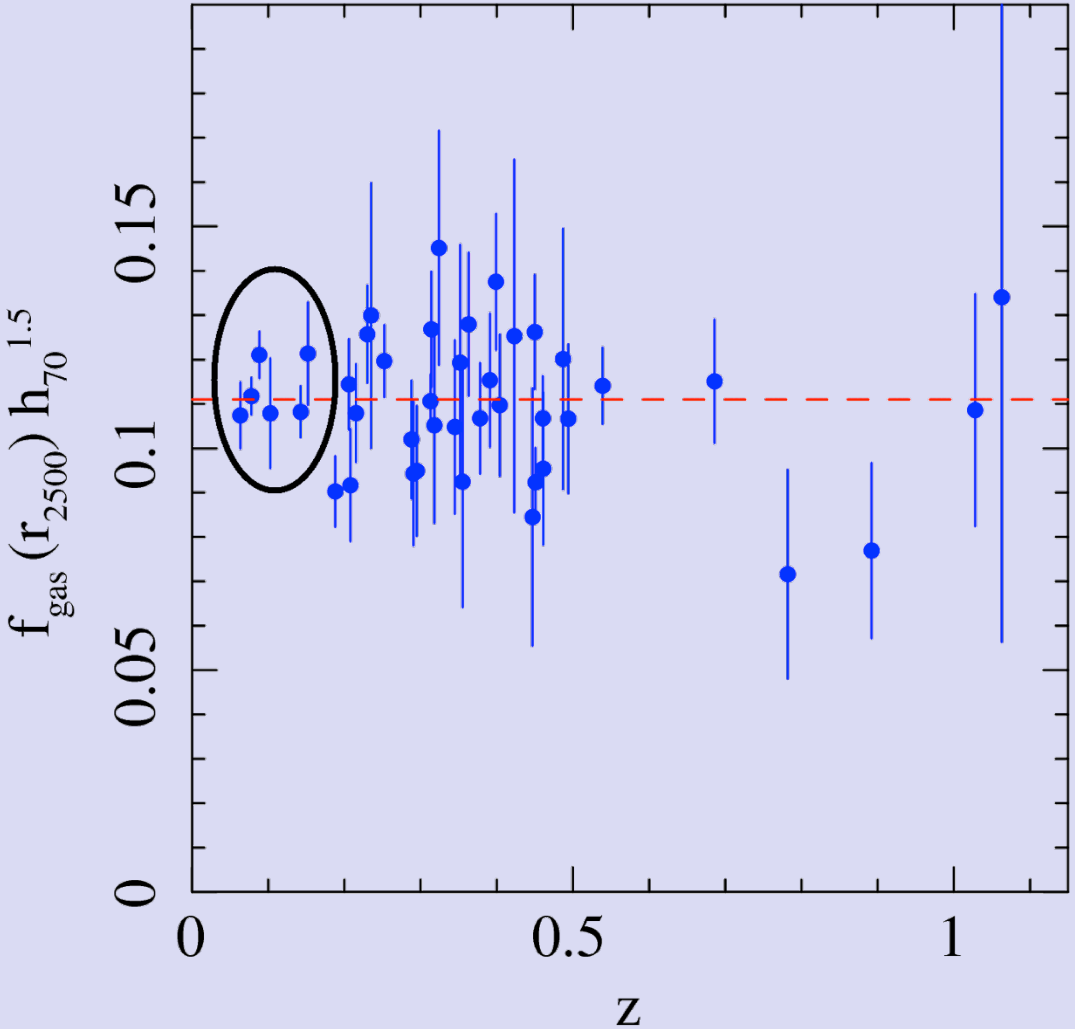
Best fit for all the data (survey+follow-up+other data).



Both, power law, self-similar, constant log-normal scatter.

* Crucial: self-consistent and simultaneous analysis of survey+follow-up data, accounting for selection biases, degeneracies, covariances, and systematic uncertainties.
* Data does not require additional evolution beyond self-similar (see tests in Mantz et al 10b).
* Important cluster astrophysics conclusions (see Mantz et al 10b).

Gas mass fraction: calibration data
Total mass proxy



ΛCDM (Ω_m=0.3, Ω_Λ=0.7)

Only the 6 lowest-z clusters

Hot kT>5keV

z<0.15

Scatter <~10% in f_{gas}

New likelihood approach: simultaneous and self-consistent

- To properly account for selection biases [in a previous analysis of the mass function, Mantz et al 08 (M08), using an external data set to constrain the luminosity-mass relation, we restrict the data set of Reiprich & Bohringer 02 to low redshift and high fluxes to minimize the effects of selection bias].

- M08, Vikhlinin et al 09a,b binned their detected clusters in redshift and mass with infinitesimally small bins taking the previous approach to its logical limit, but there was still no self-consistent fit for both scaling relations and cosmology.

- Generalization of M08 to allow a simultaneous and self-consistent fit using follow-up observations of flux-selected clusters over the whole redshift range of the data accounting for both Malmquist and Eddington biases.

- Likelihood can be derived from first principles beginning from a Bayesian regression model.

- General problem: counting sources as a function of their properties

New likelihood approach: simultaneous and self-consistent

- A population function: <dN/dx> theoretical prediction of the distribution (i.e. number) of sources as a function of their properties.

- Population variables x (properties).

- Response variables y obeying a stochastic scaling relation as a function of x.

- Stochastic scaling relation P(y|x): probability distribution of y given x.

- Observed values $\hat{x}$ and $\hat{y}$ (note that not all x and y need to be measured, except for those determining if a source belongs to the sample, i.e. if it is detected).

- Sampling distributions for the observations as a function of the population and response variables $P(\hat{x}, \hat{y} \mid x, y)$.

- A selection function $P(I \mid x, y, \hat{x}, \hat{y})$, where *I* represents the inclusion in the sample, i.e. detection.

New likelihood approach: simultaneous and self-consistent

- For our large sky coverage surveys of massive clusters we assume that the clustering of the sources is not important compared with the purely Poisson probability distribution of their occurrence (Hu & Kravtsov 03; Holder 06).

- Binning derivation: We divide the observed space $(\hat{x}, \hat{y})$ into infinitesimal bins which contain at a maximum one detected source and the population function and scaling relations are assumed to be constant in each bin.

Expected number of detected sources

$$\langle N_{\mathrm{det},j} \rangle = (\Delta \hat{x}_j \Delta \hat{y}_j) \int \mathrm{d}x \int \mathrm{d}y \left\langle \frac{\mathrm{d}N}{\mathrm{d}x} \right\rangle P(y|x) P(\hat{x}_j, \hat{y}_j | x, y) P(I | x, y, \hat{x}_j, \hat{y}_j)$$

Likelihood (product of Poisson likelihoods)

$$\mathcal{L}(\{N_j\}) = \prod_j \frac{\langle N_{\mathrm{det},j} \rangle^{N_j} \mathrm{e}^{-\langle N_{\mathrm{det},j} \rangle}}{N_j!} \underset{\substack{\uparrow \\ N_j \in \{0,1\}}}{=} \mathrm{e}^{-\langle N_{\mathrm{det}} \rangle} \prod_{j: N_j = 1} \langle N_{\mathrm{det},j} \rangle$$

New likelihood approach: simultaneous and self-consistent

- Regression derivation: for truncated data (with undetected sources) the total number of sources is the addition of detected plus undetected (missed) sources and is part of the model and must be marginalized over (Gelman et al 04; Kelly 07).

$$\langle N \rangle = \int \mathrm{d}x \left\langle \frac{\mathrm{d}N}{\mathrm{d}x} \right\rangle$$

$$\langle N_{\mathrm{det}} \rangle = \int \mathrm{d}x \left\langle \frac{\mathrm{d}N}{\mathrm{d}x} \right\rangle \int \mathrm{d}y \; P(y|x) \int \mathrm{d}\hat{x} \int \mathrm{d}\hat{y} \; P(\hat{x}, \hat{y}|x, y) P(I|x, y, \hat{x}, \hat{y})$$

$$\langle N_{\mathrm{mis}} \rangle = \langle N \rangle - \langle N_{\mathrm{det}} \rangle$$

- Joint likelihood of the observations $(\hat{x}, \hat{y})$ and the total number of sources N is

Number of ways of selecting N_{det} from N

Probability of not detecting N_{mis}

$$\mathcal{L}(\hat{x}, \hat{y}, N) = \left[\frac{\langle N \rangle^N \mathrm{e}^{-\langle N \rangle}}{N!} \right] \left[\frac{N!}{N_{\mathrm{det}}! N_{\mathrm{mis}}!} \right] \prod_{i=1}^{N_{\mathrm{det}}} P_{\mathrm{det}}(\hat{x}_i, \hat{y}_i, I) \prod_{j=1}^{N_{\mathrm{mis}}} P_{\mathrm{mis}}(\bar{I})$$

Poisson likelihood for N

Probability that the N_{det} have measurements $(\hat{x}, \hat{y})$

New likelihood approach: simultaneous and self-consistent

- Using the previous expressions we can calculate the probabilities of detecting a source with given properties and of missing a source

$$P_{\mathrm{det}}(\hat{x}_i, \hat{y}_i, I) = \int \mathrm{d}x \int \mathrm{d}y \frac{\langle \mathrm{d}N/\mathrm{d}x \rangle}{\langle N \rangle} P(y|x)\, P(\hat{x}_i, \hat{y}_i | x, y) P(I | x, y, \hat{x}_i, \hat{y}_i) = \frac{\langle \tilde{n}_{\mathrm{det},i} \rangle}{\langle N \rangle}$$

$P(x)$ Probability for a source to have properties x

$$P_{\mathrm{mis}}(\bar{I}) = \int \mathrm{d}x \int \mathrm{d}y \frac{\langle \mathrm{d}N/\mathrm{d}x \rangle}{\langle N \rangle} P(y|x) \int \mathrm{d}\hat{x} \int \mathrm{d}\hat{y}\; P(\hat{x}, \hat{y} | x, y) P(\bar{I} | x, y, \hat{x}, \hat{y}) = \frac{\langle N_{\mathrm{mis}} \rangle}{\langle N \rangle}$$

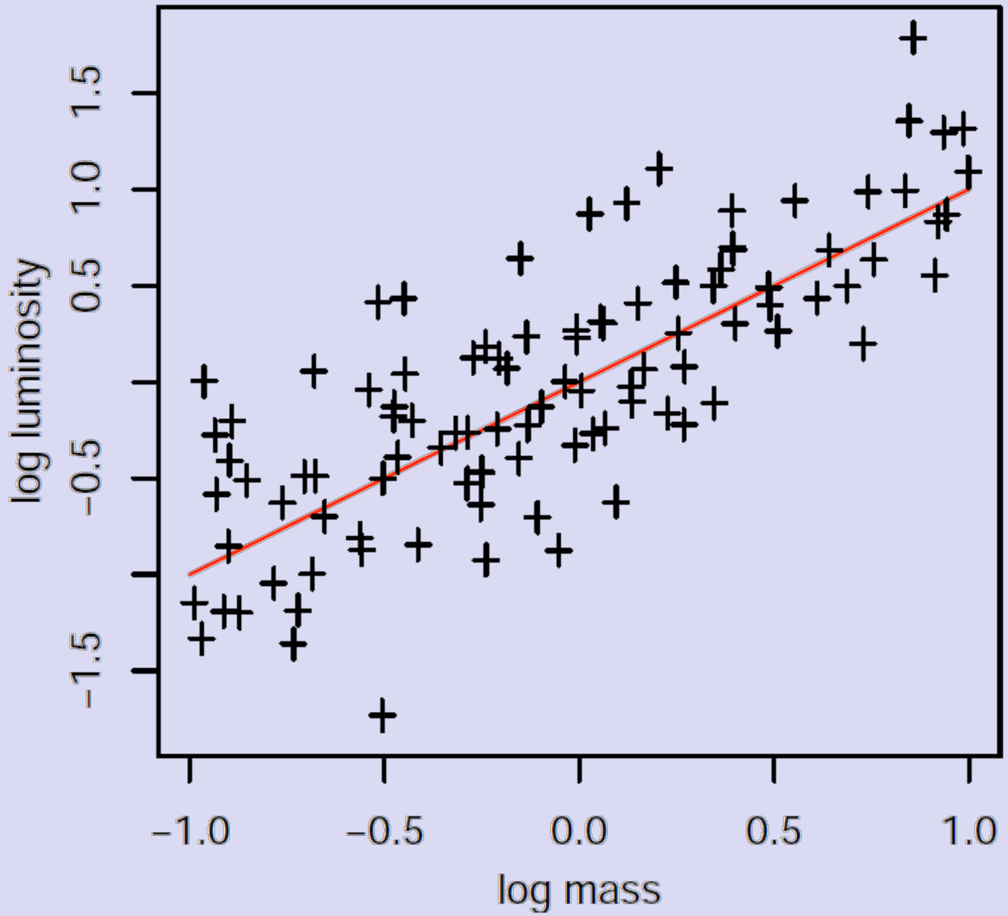
Substituting these expressions we have

$$\mathcal{L}(\hat{x}, \hat{y}, N) = \left[\frac{\langle N \rangle^{N}}{\langle N \rangle^{N_{\mathrm{det}}} \langle N \rangle^{N_{\mathrm{mis}}}} \right] \left[\frac{1}{N_{\mathrm{det}}!} \right] \left[\frac{\langle N_{\mathrm{mis}} \rangle^{N_{\mathrm{mis}}}\; \mathrm{e}^{-\langle N_{\mathrm{mis}} \rangle}}{N_{\mathrm{mis}}!} \right] \mathrm{e}^{-\langle N_{\mathrm{det}} \rangle} \prod_{i=1}^{N_{\mathrm{det}}} \langle \tilde{n}_{\mathrm{det},i} \rangle$$

$$\langle \tilde{n}_{\mathrm{det},j} \rangle = \langle N_{\mathrm{det},j} \rangle / (\Delta \hat{x}_j \Delta \hat{y}_j)$$

Luminosity-mass scaling relation: selection biases

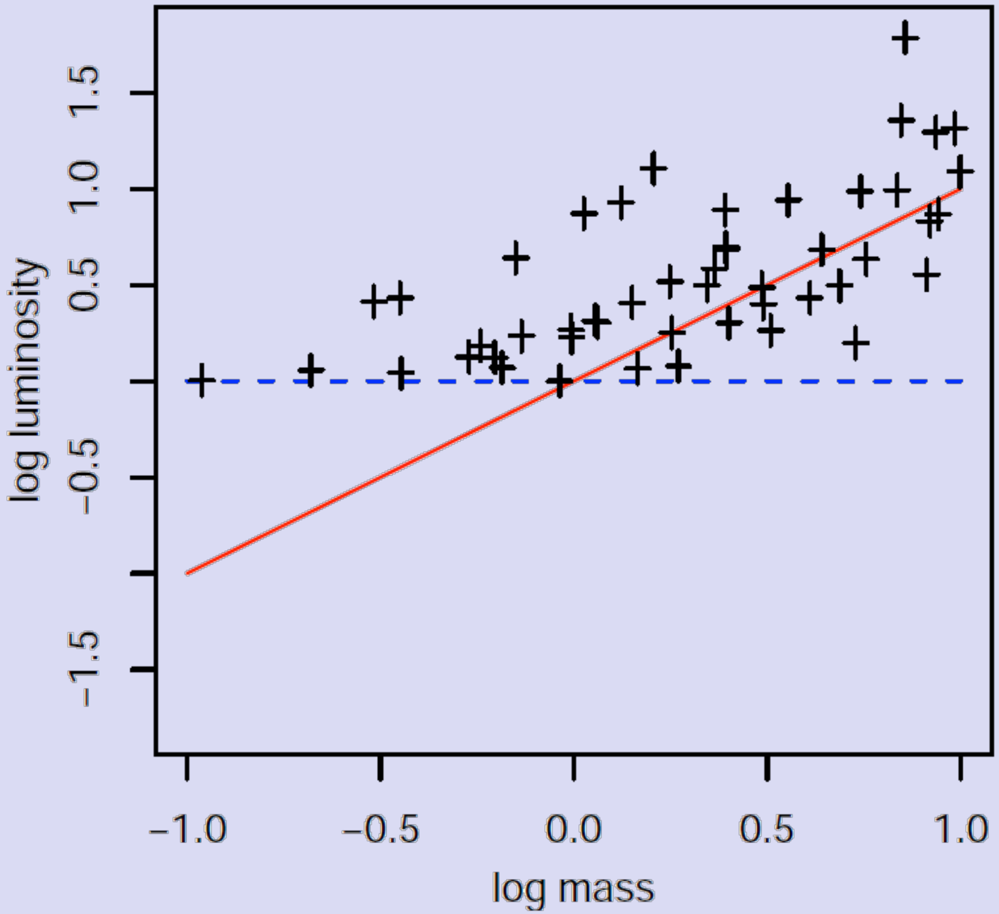
Mantz et al 10b



For illustration purposes: Uniform distribution of simulated data and fictitious luminosity-mass relation (red line).

Luminosity-mass scaling relation: selection biases

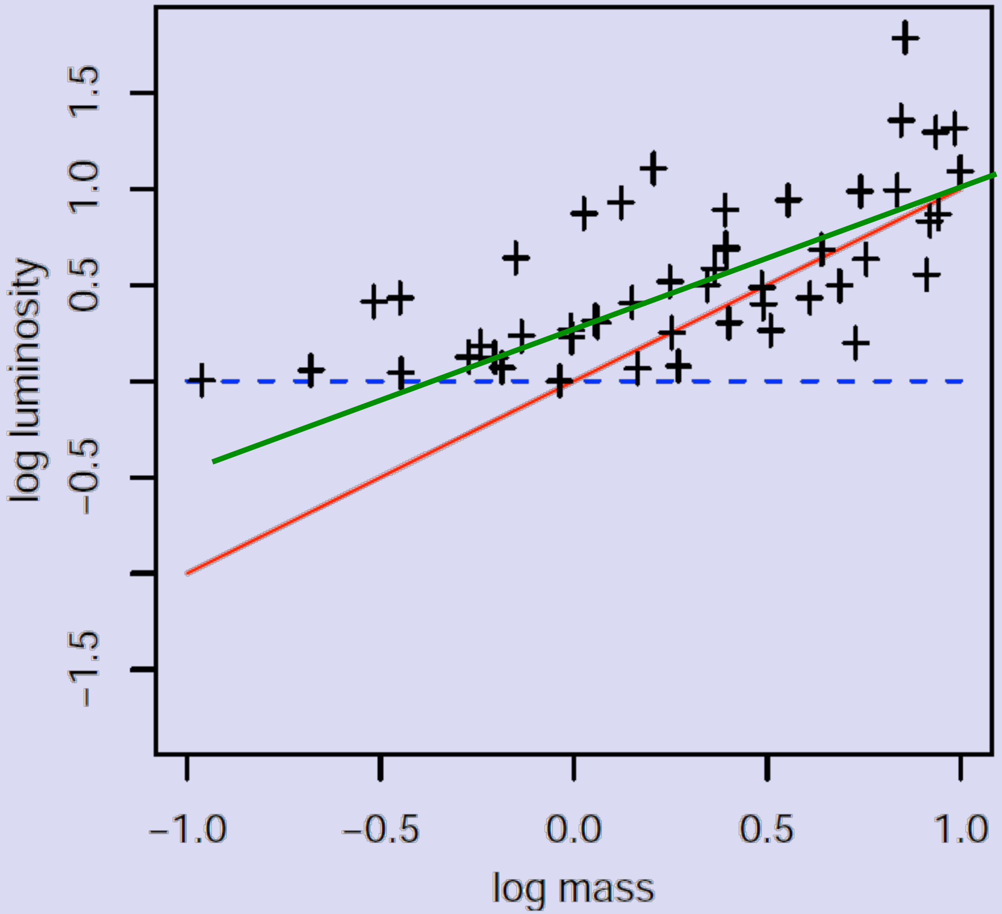
Mantz et al 10b



For illustration purposes: Uniform distribution of simulated data and fictitious luminosity-mass relation (red line).

* The luminosity-mass relation has intrinsic scatter (~40%), which leads to Malmquist bias: brighter cluster are easier to find.

Luminosity-mass scaling relation: selection biases



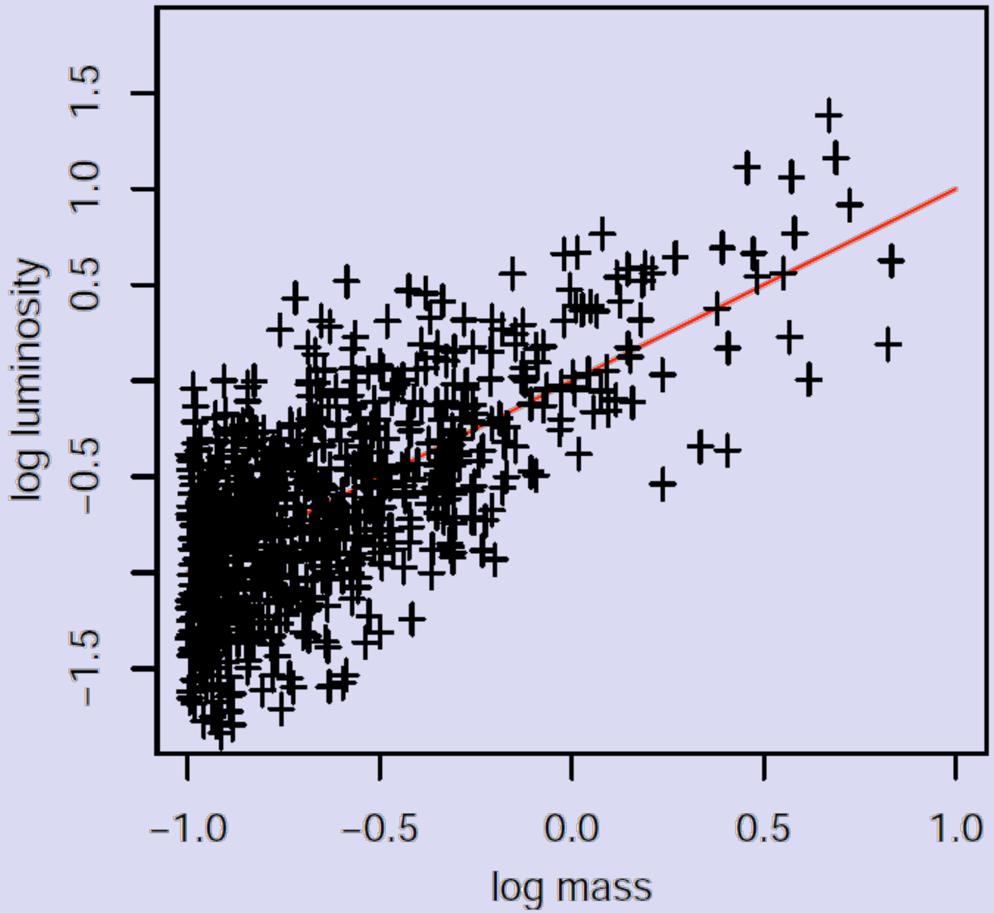
For illustration purposes: Uniform distribution of simulated data and fictitious luminosity-mass relation (red line).

* The luminosity-mass relation has intrinsic scatter (~40%), which leads to Malmquist bias: brighter cluster are easier to find.

* For illustration purposes: fitting by eye (green line) only these data is wrong.

Luminosity-mass scaling relation: selection biases

Mantz et al 10b



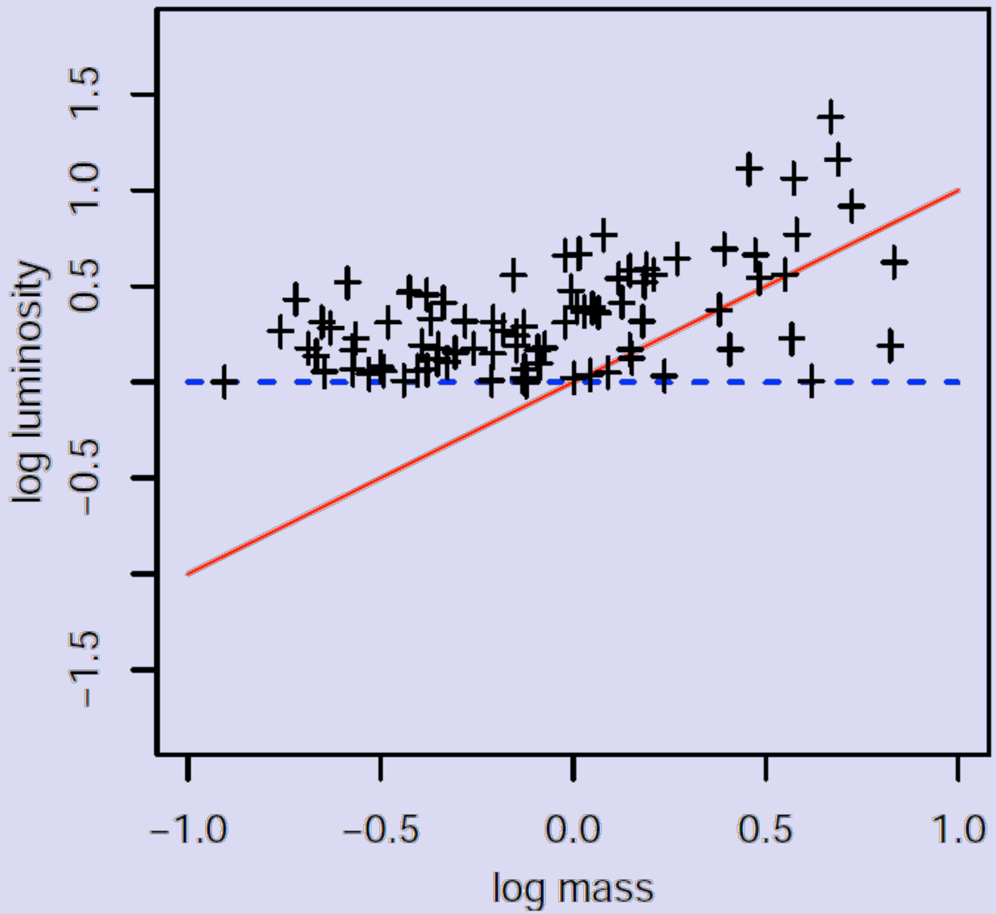
For illustration purposes: Exponential distribution of simulated data and fictitious luminosity-mass relation (red line).

* The luminosity-mass relation has intrinsic scatter (~40%), which leads to Malmquist bias: brighter cluster are easier to find.

* The shape of the mass function leads to Eddington bias: much more low-mass clusters

Luminosity-mass scaling relation: selection biases

Mantz et al 10b



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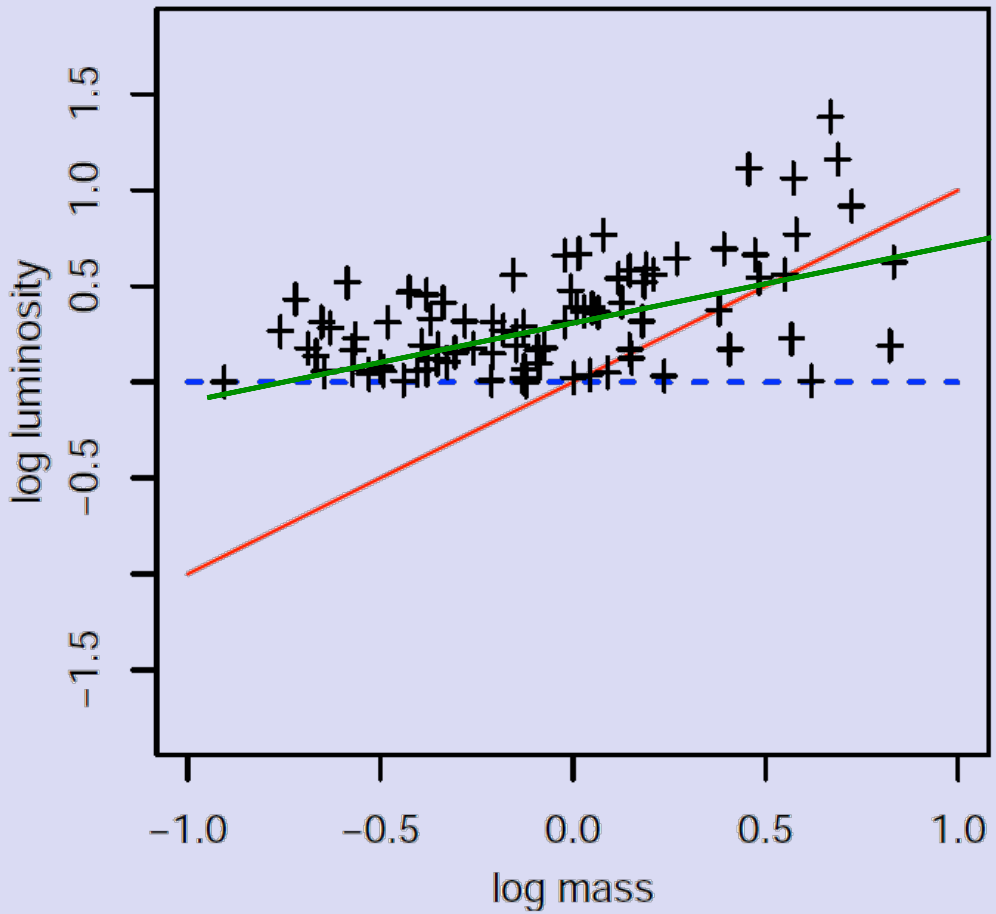
* The shape of the mass function leads to Eddington bias: much more low-mass clusters

Luminosity-mass scaling relation: selection biases



For illustration purposes: Exponential distribution of simulated data and fictitious luminosity-mass relation (red line).

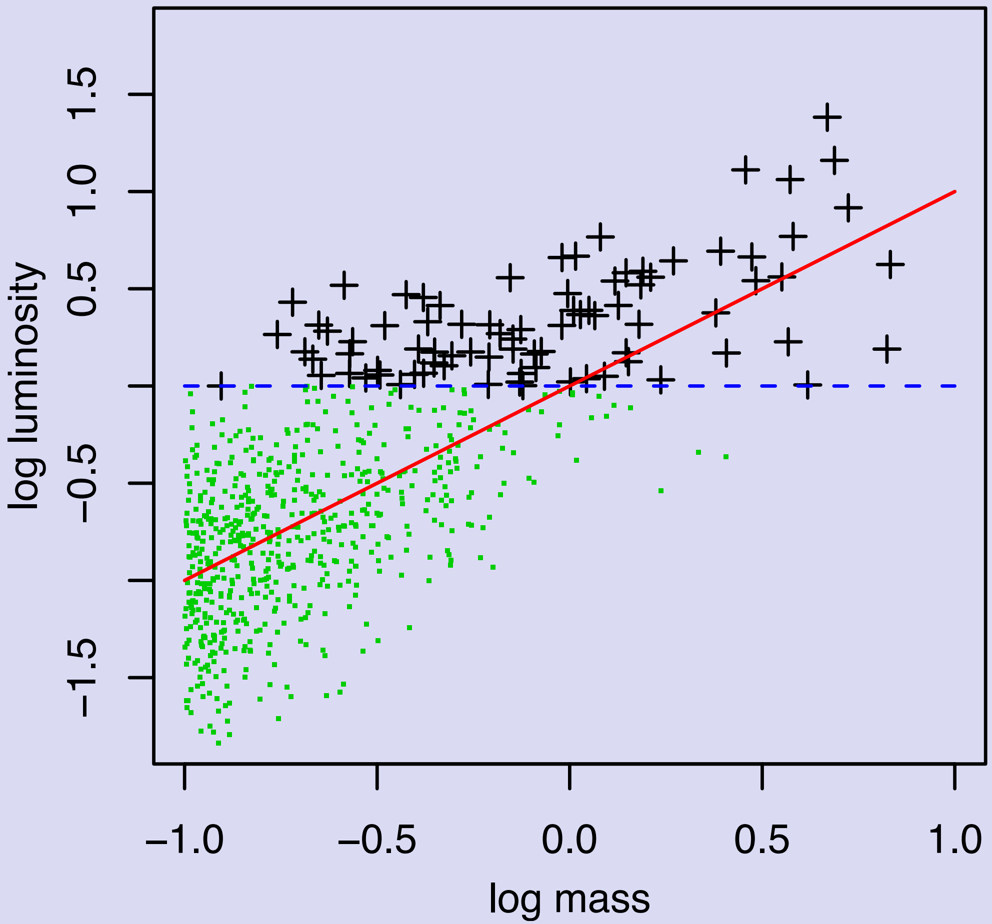
* The luminosity-mass relation has intrinsic scatter (~40%), which leads to Malmquist bias: brighter cluster are easier to find.

* The shape of the mass function leads to Eddington bias: much more low-mass clusters.

* For illustration purposes: fitting by eye (green line) only these data is wrong.

Luminosity-mass scaling relation: selection biases

Allen, Evrard, Mantz 11

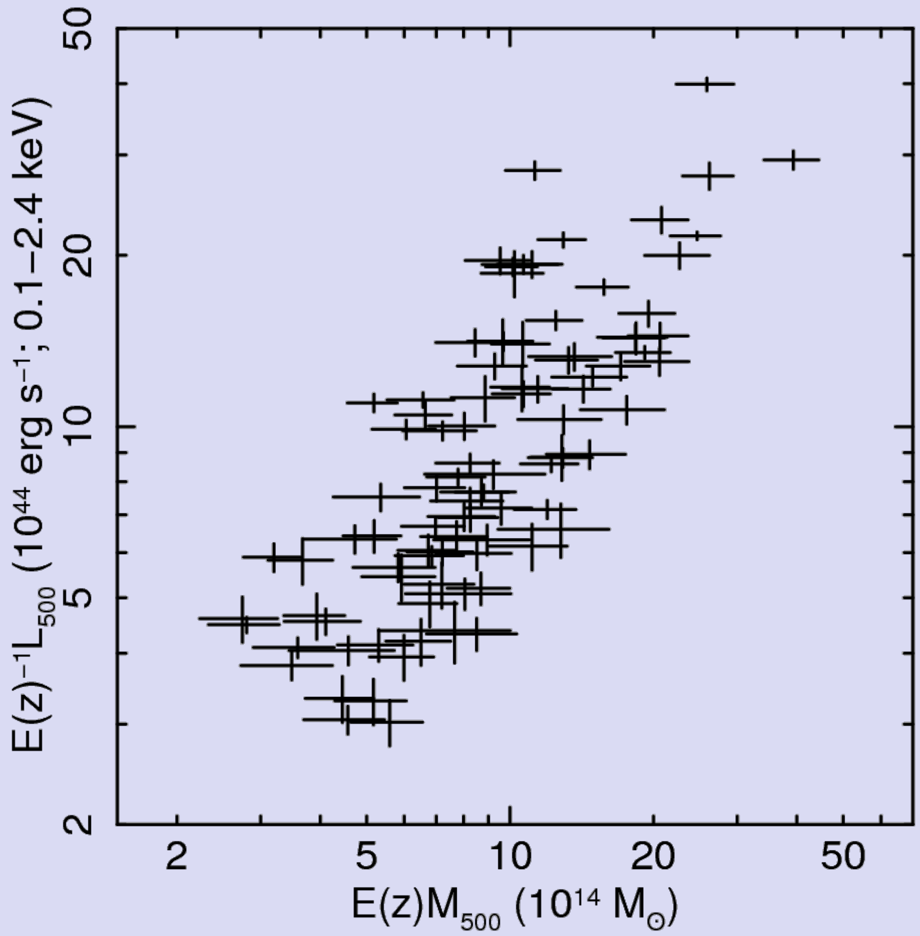


For illustration purposes: Exponential distribution of simulated data and fictitious luminosity-mass relation (red line).

* The luminosity-mass relation has intrinsic scatter (~40%), which leads to Malmquist bias: brighter cluster are easier to find.

* The shape of the mass function leads to Eddington bias: much more low-mass clusters

X-ray luminosity-mass relation



Fitted with simple power law model, self-similar evolution and constant log-normal scatter σ_{lm}

$$\langle l(m) \rangle = \beta_0^{lm} + \beta_1^{lm} m$$

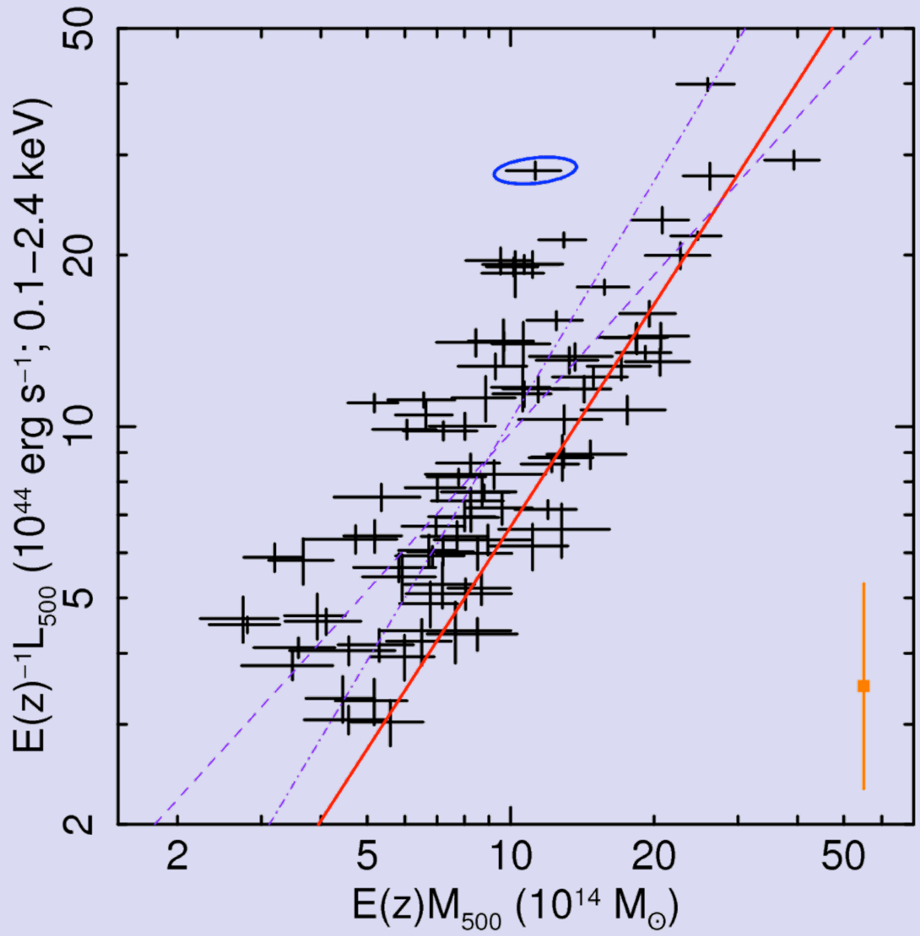
Using the definitions

$$l = \log_{10}\left(\frac{L_{500}}{E(z)10^{44}\, erg\, s^{-1}} \right)$$

$$m = \log_{10}\left(\frac{M_{500}E(z)}{10^{15}\, M_{solar}} \right)$$

Current data do not require (i.e. acceptable fit) neither additional evolution beyond self-similar and constant scatter or asymmetric scatter (see details in Mantz et al 10b).

X-ray luminosity-mass relation



For bolometric luminosities, the best fit using all the data (survey+follow-up+other cosmological data sets):

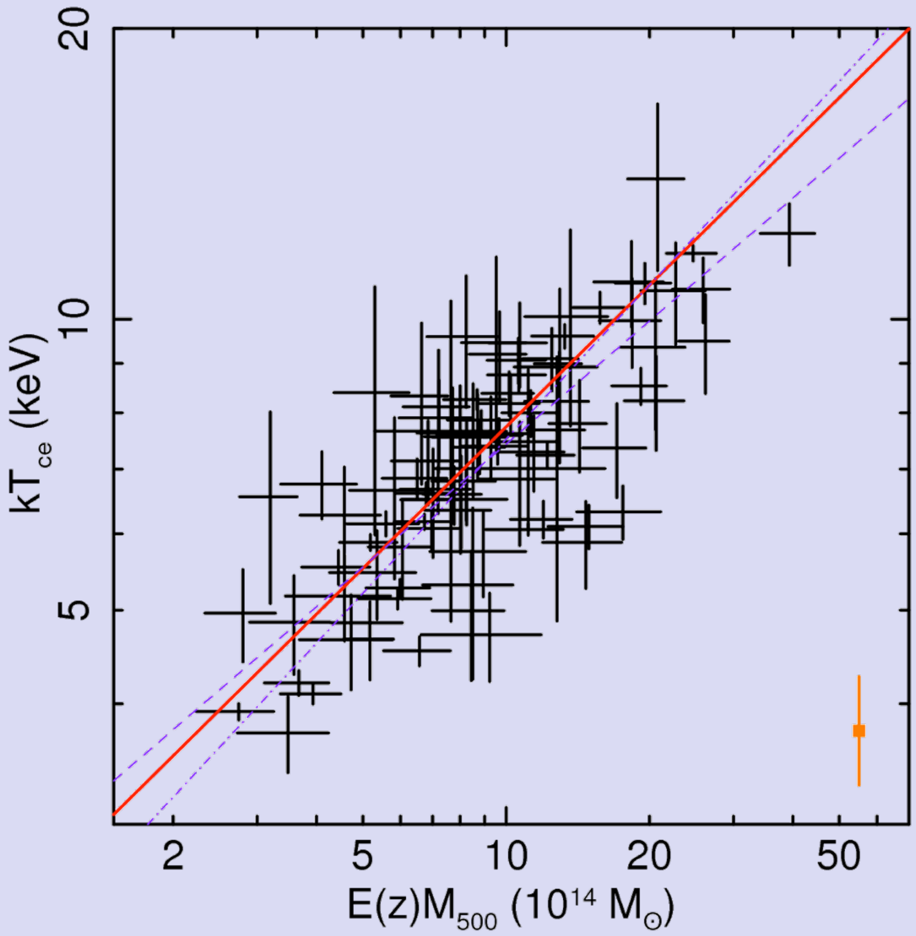
norm.	$\beta_0^{lm} = 1.23 \pm 0.12$
slope	$\beta_1^{lm} = 1.63 \pm 0.06$
scatter	$\sigma_{lm} = 0.185 \pm 0.019$ (~ 40%)

Slope steeper than the simple virial prediction: $\beta_1^{lm} = 1.33$

Consistent with excess heating

Energy injection heats (e.g. AGN) the gas raising the temperature, decreasing the density and therefore the luminosity, being more important for less massive systems.

Temperature-mass relation



Again, simple power law, self-similar, constant log-normal scatter. Best fit for all the data:

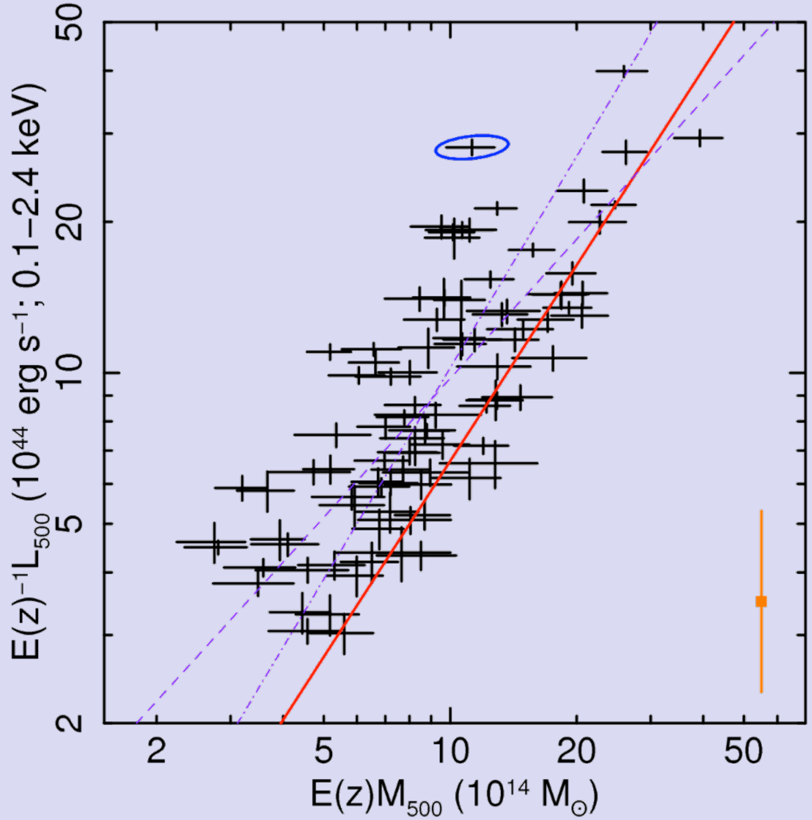
norm.	$\beta_0^{tm} = 0.89 \pm 0.03$
slope	$\beta_1^{tm} = 0.49 \pm 0.04$
scatter	$\sigma_{tm} = 0.055 \pm 0.008 \ (\sim 15\%)$

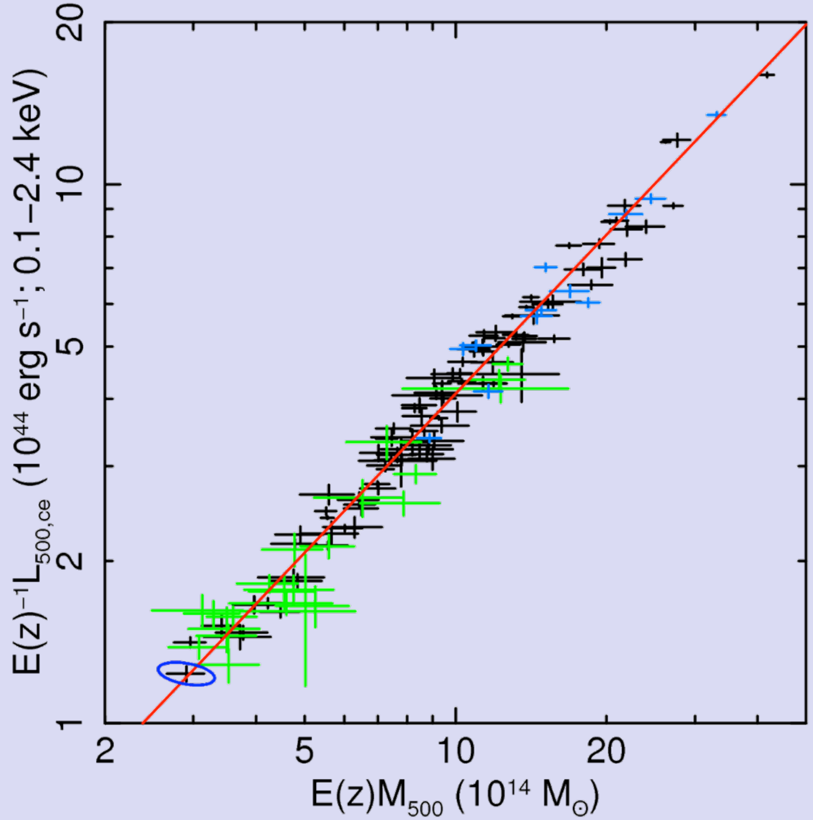
Slope shallower than the simple virial prediction: $\beta_1^{tm} = 0.67$

Consistent with excess heating

Energy injection heats (e.g. AGN) the gas raising the temperature, decreasing the density and therefore the luminosity, being more important for less massive systems.

X-ray luminosity-mass relation





Core-included: scatter ~40%

Data consistent with self-similar evolution suggesting that excess heating occurred at z>0.5

Core-excised $r<0.15r_{500}$.

Scatter undetected <5%.

$\beta_1^{lm} = 1.30 \pm 0.05$ Consistent with the virial th.

Excess heating limited to the centers / effective mass-limited cluster sample could be possible

Sampling model: follow-up observations

$$M(r_{500}) = \frac{M_{\rm gas}(r_{500})}{f_{\rm gas}(r_{500})} = \frac{4\pi}{3}(500)\rho_{\rm cr}(z)r_{500}^3$$

Masses, luminosities and temperatures measured at r_{500}

$$M_{\rm gas}(r) \propto \rho_{\rm cr}(z)r^3 f_{\rm gas}(r) \propto r^{\eta_g}$$

$$\eta_g = 1.092 \pm 0.006$$

n_g logarithmic slope of the gas mass profiles at large radius; fit to the entire sample from 0.7-1.3r_{500}

$$r_{500} \propto \left[f_{\rm gas}(r_{500})H^2(z)\right]^{1/(\eta_g-3)}$$

$$\frac{M^{\rm ref}(r)}{M(r)} = \frac{M_{\rm gas}^{\rm ref}(r)/f_{\rm gas}^{\rm ref}(r)}{M_{\rm gas}(r)/f_{\rm gas}(r)}R_{\rm NFW} = \frac{d_A^{\rm ref}(z)^{2.5} f_{\rm gas}}{d_A(z)^{2.5} f_{\rm gas}^{\rm ref}}R_{\rm NFW}$$

We assume that the NFW profile is a good approximation here

$$L_{500}(r) \propto d_{\rm L}^2(z)\left(\frac{r_{500}}{d_A(z)}\right)^{\eta_{\rm L}}$$

n_L=0.1135+-0.0005

Theory: linear and non-linear

$$n(M,z) = \int_0^M f(\sigma)\,\frac{\bar{\rho}_\mathrm{m}}{M'}\,\frac{\mathrm{d}\ln\sigma^{-1}}{\mathrm{d}M'}\,\mathrm{d}M'$$

Number density of galaxy clusters

$$\sigma^2(M,z) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k,z)|W_\mathrm{M}(k)|^2\,\mathrm{d}k$$

Variance of the density fluctuations

$$P(k,z) \propto k^{n_\mathrm{s}} T^2(k,z_\mathrm{t})\,D(z)^2$$

Linear power spectrum

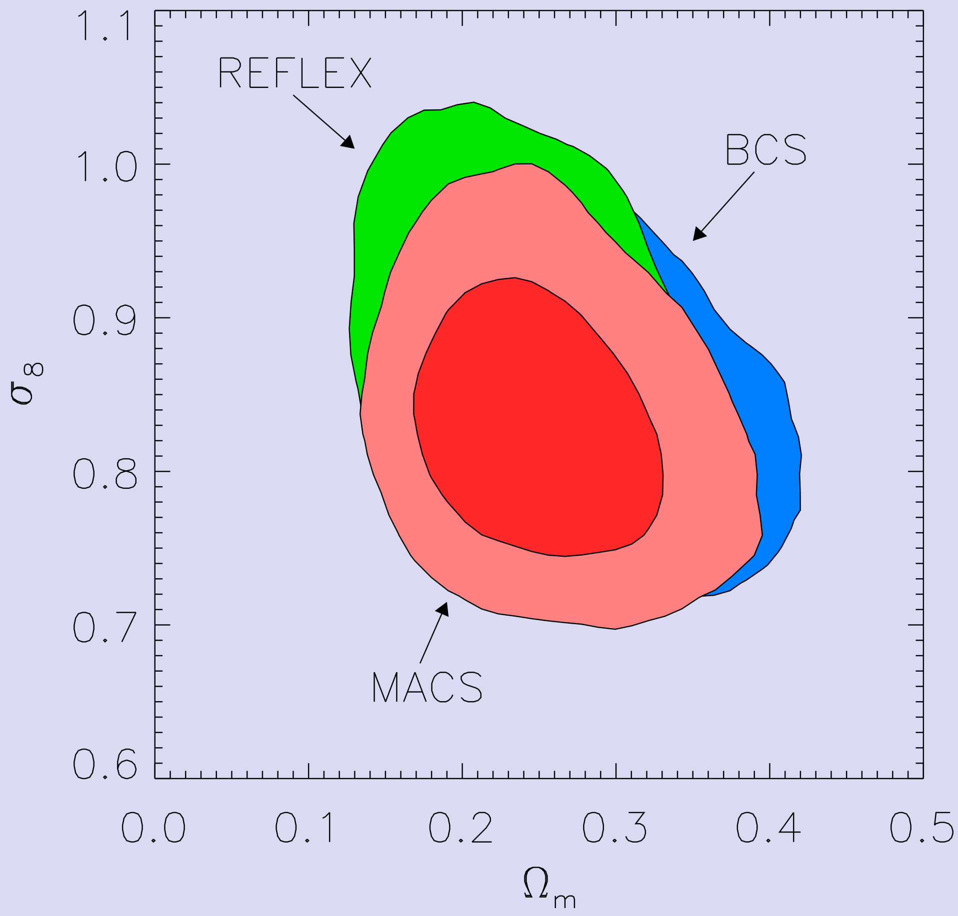
$$f(\sigma,z) = A\left[\left(\frac{\sigma}{b}\right)^{-a}+1\right]\mathrm{e}^{-c/\sigma^2}$$

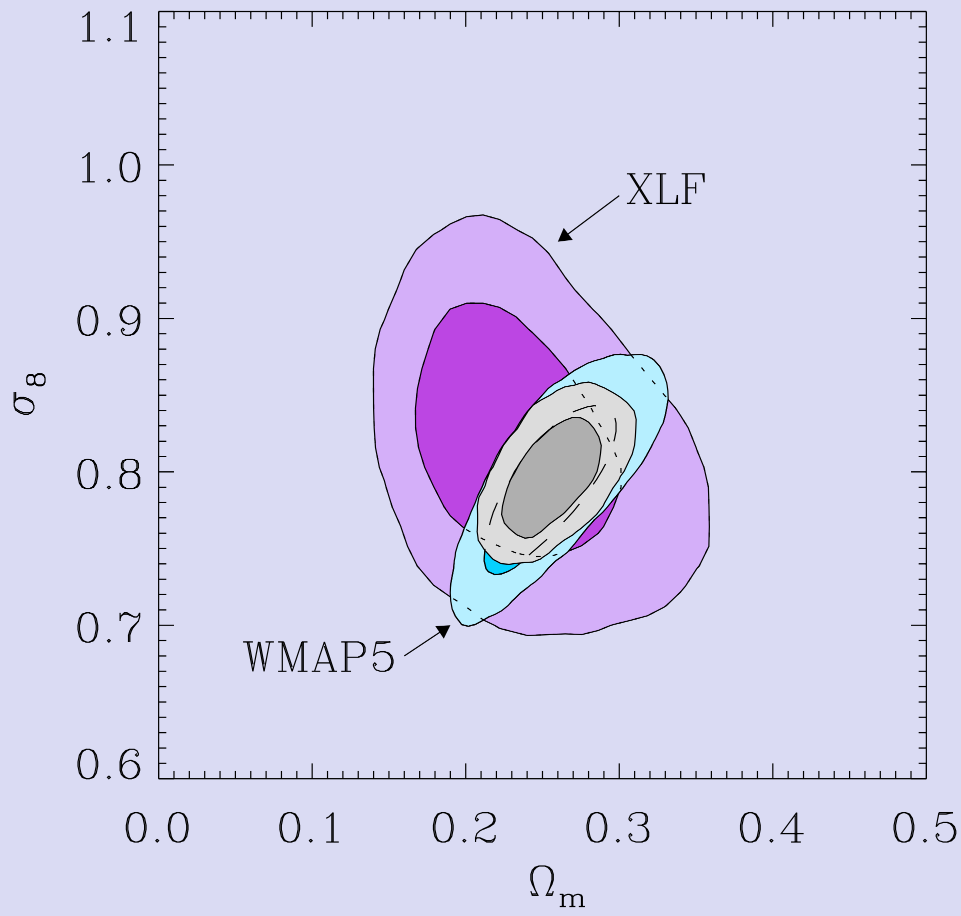
Fitting formula from N-body simulations (Tinker et al 08)

$$x(z) = x_0(1+z)^{\varepsilon\alpha_x} \qquad x \in \{A, a, b, c\}$$

Flat ΛCDM

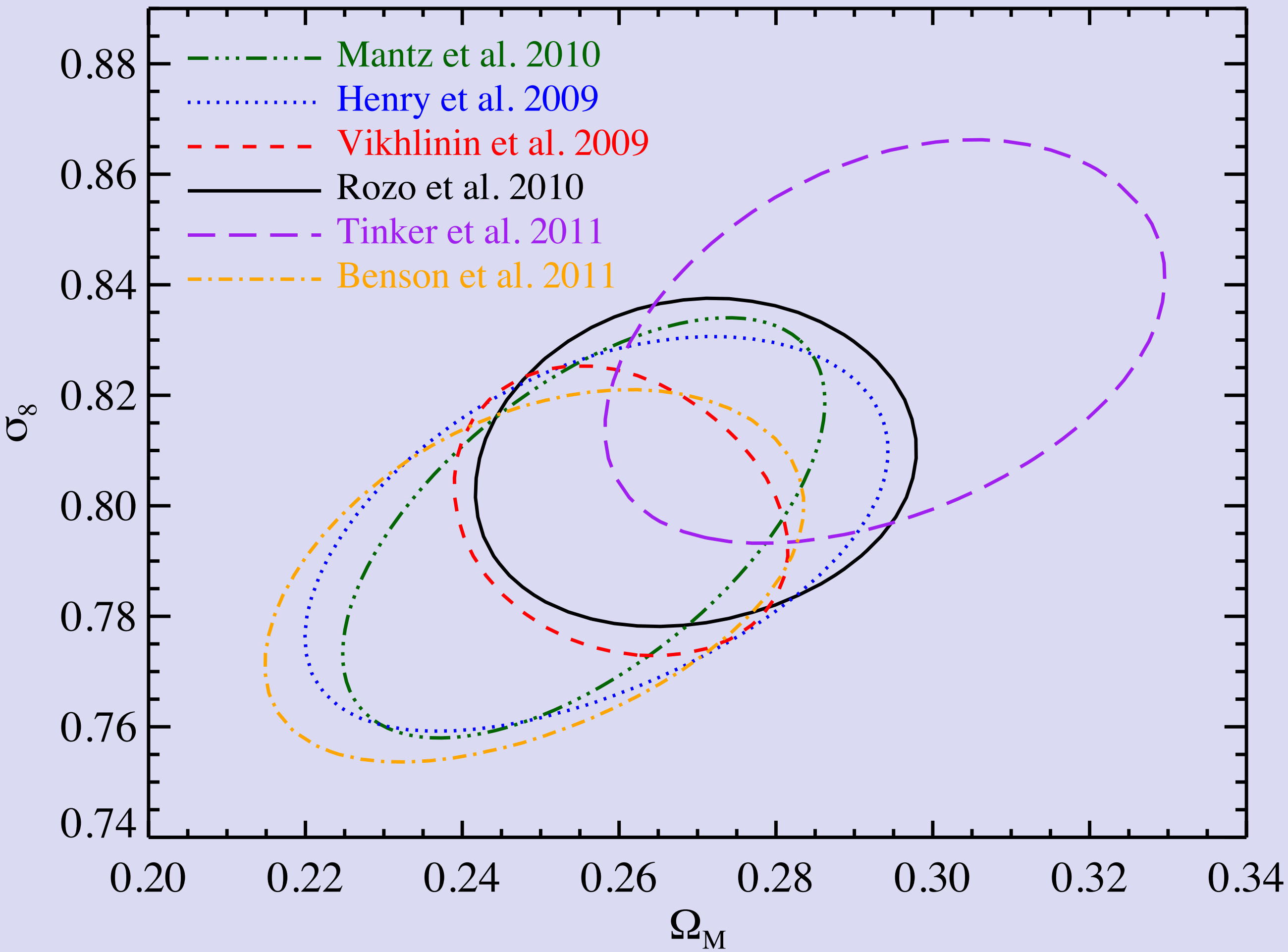
Mantz et al 10a





Agreement between cluster experiments

From Weinberg et al 12



Constraints on dark energy

All data sets

1. Abundance of massive clusters (X-ray Luminosity Function, XLF) to measure cosmic expansion and growth of matter fluctuations with respect to the mean density.

$$D(z) \equiv \frac{\delta(z)}{\delta(z_{\rm t})} = \frac{\sigma(M,z)}{\sigma(M,z_{\rm t})} \qquad \delta = (\rho_{\rm m} - \bar{\rho}_{\rm m})/\bar{\rho}_{\rm m}$$

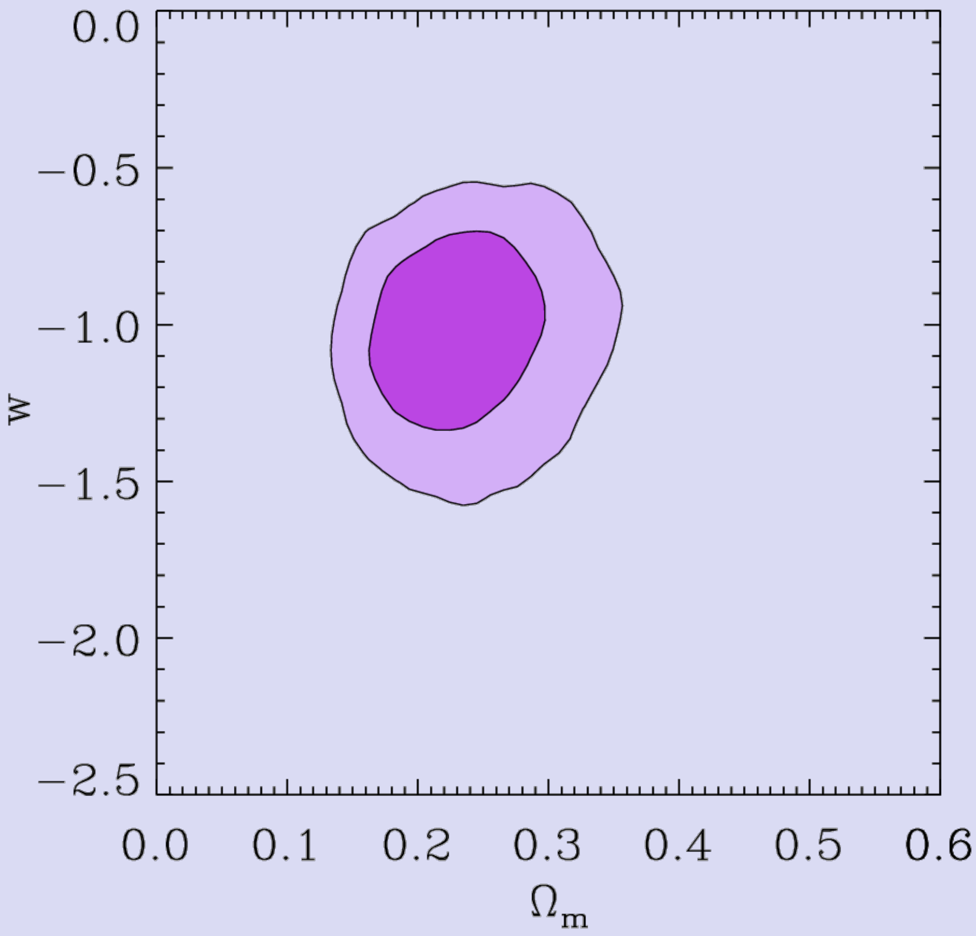
2. SNIa, f_{gas}, XLF, CMB, BAO to measure the cosmic expansion of the background density. We use three expansion histories well fitted by these data sets.

$$E(a) = \left[\Omega_{\rm m}\, a^{-3} + \Omega_{\rm de}\, a^{-3(1+w)} + \Omega_{\rm k}\, a^{-2}\right]^{1/2}$$

i) flat ΛCDM w=-1, Ω_k=0
ii) flat *w*CDM w constant, Ω_k=0
iii) non-flat ΛCDM w=-1, Ω_k constant

Dark Energy results: flat wCDM

Mantz et al 10a

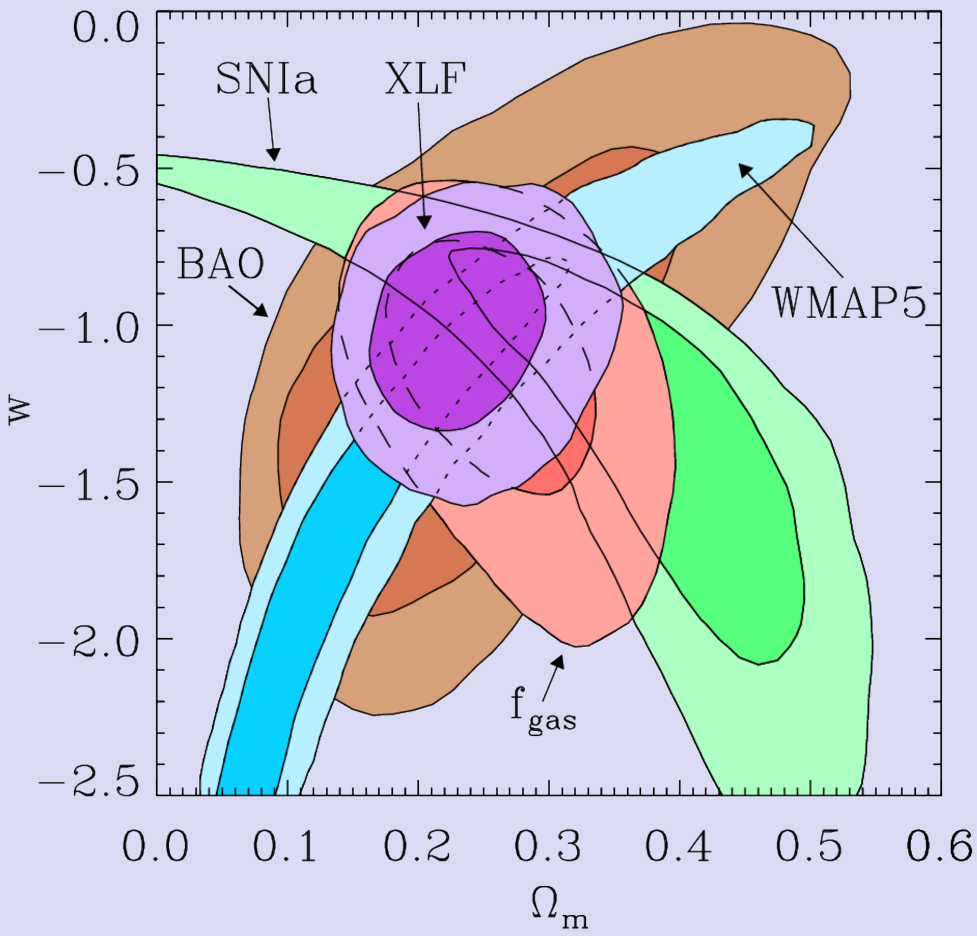


XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238 clusters (Mantz et al 10a). Including systematics.

Ω_m = 0.23 +- 0.04
σ_8 = 0.82 +- 0.05
w = -1.01 +- 0.20

Dark Energy results: flat wCDM

Mantz et al 10a



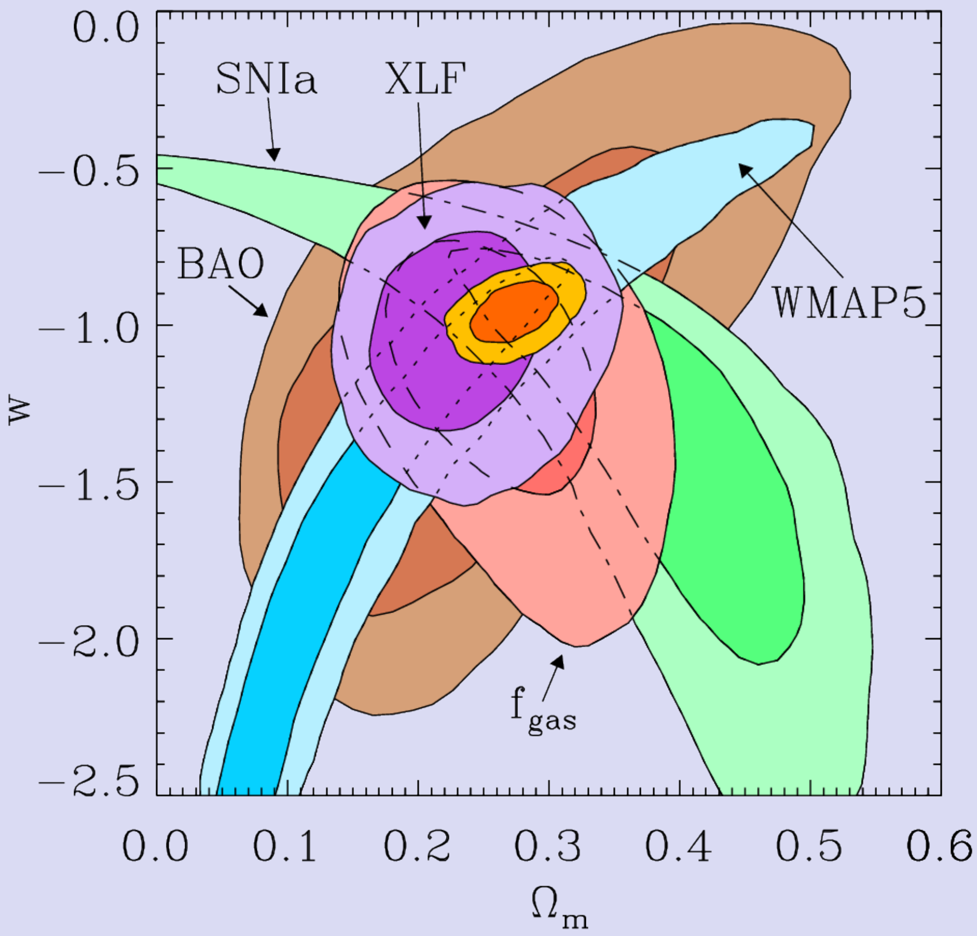
Green: SNIa (Kowalski et al 08, Union)
Blue: CMB (WMAP5)
Red: cluster f_{gas} (Allen et al 08)
Brown: BAO (Percival et al 07)

XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238 clusters (Mantz et al 10a). Including systematics

Ω_m = 0.23 +- 0.04
σ_8 = 0.82 +- 0.05
w = -1.01 +- 0.20

Dark Energy results: flat wCDM

Mantz et al 10a



Good mass proxy at all z

Green: SNIa (Kowalski et al 08, Union)

Blue: CMB (WMAP5)

Red: cluster f_{gas} (Allen et al 08)

Brown: BAO (Percival et al 07)

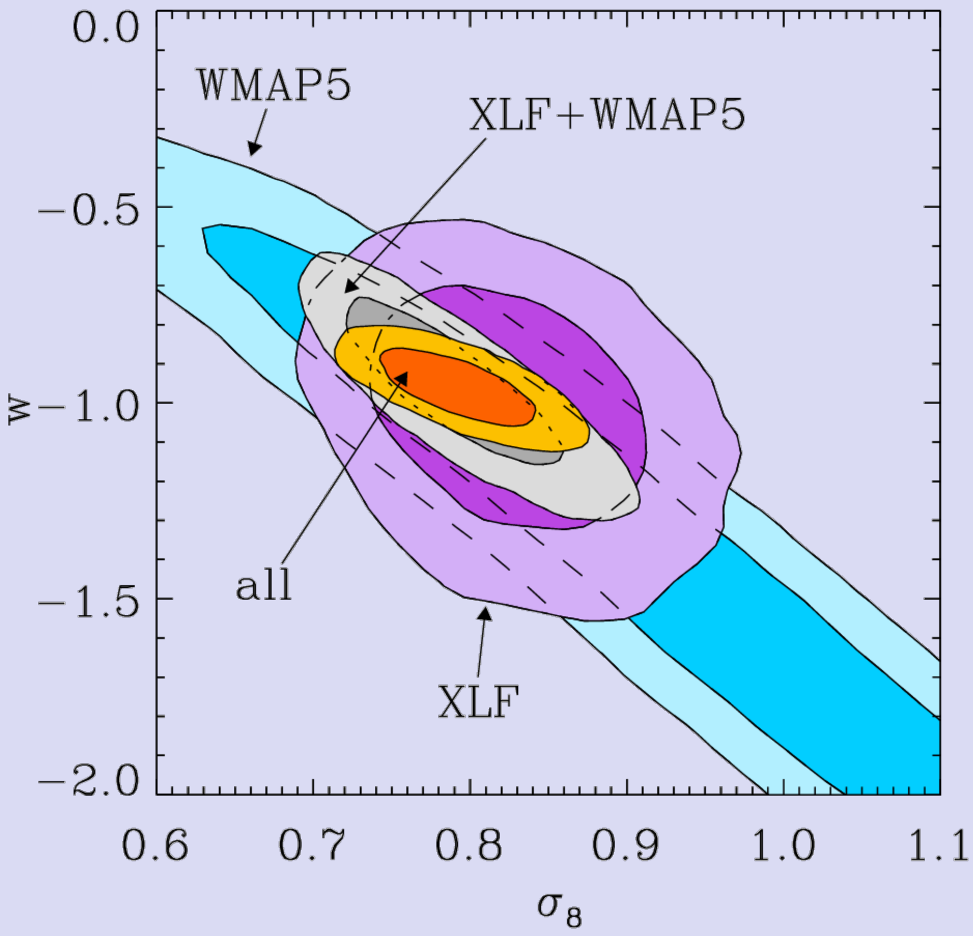
Gold: XLF+f_{gas}+WMAP5+SNIa+BAO

XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238 clusters (Mantz et al 10a). Including systematics

Ω_m = 0.23 +- 0.04
σ_8 = 0.82 +- 0.05
w = -1.01 +- 0.20

Dark Energy results: flat wCDM

Mantz et al 10a



Grey: XLF+WMAP5
Blue: CMB (WMAP5)
Gold: XLF+f_{gas}+WMAP5+SNIa+BAO

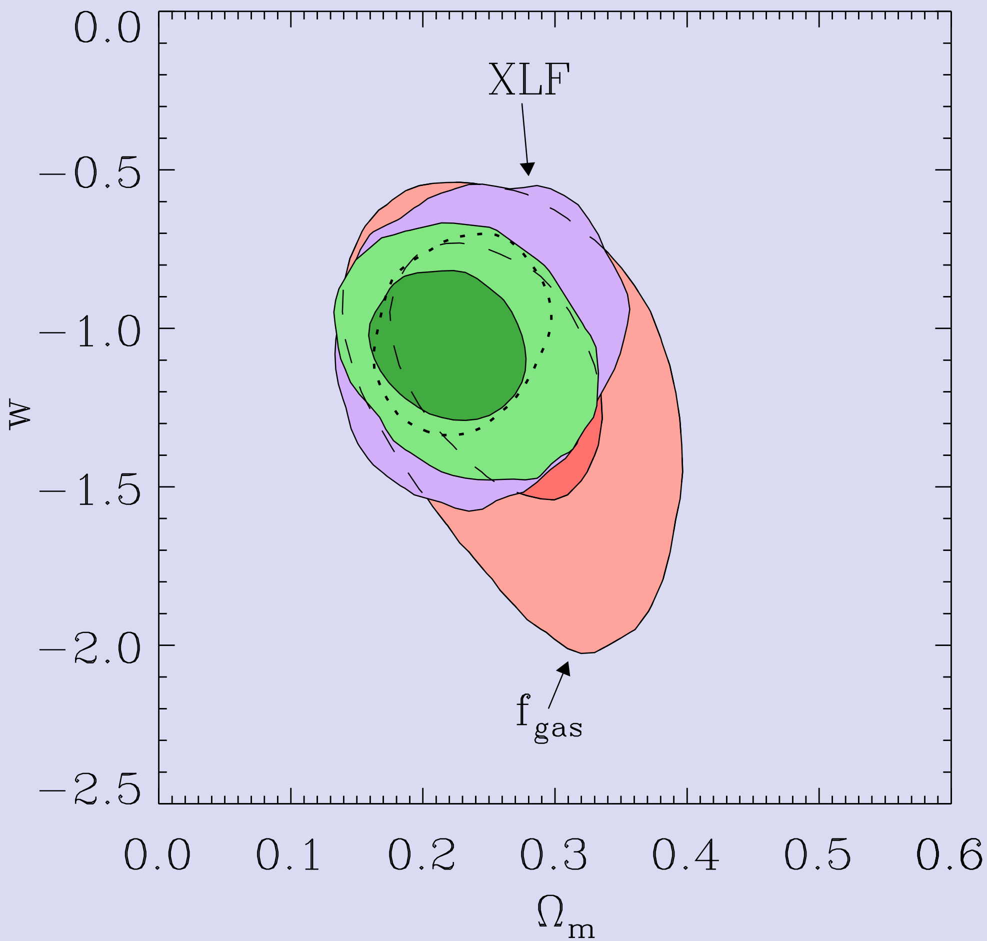
Ω_m = 0.272 +- 0.016
σ_8 = 0.79 +- 0.03
w = -0.96 +- 0.06

XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238 clusters (Mantz et al 10a). Including systematics

Ω_m = 0.23 +- 0.04
σ_8 = 0.82 +- 0.05
w = -1.01 +- 0.20

Dark Energy results: flat wCDM

Allen, Evrard & Mantz 11



Red: cluster f_{gas} (Allen et al 08)

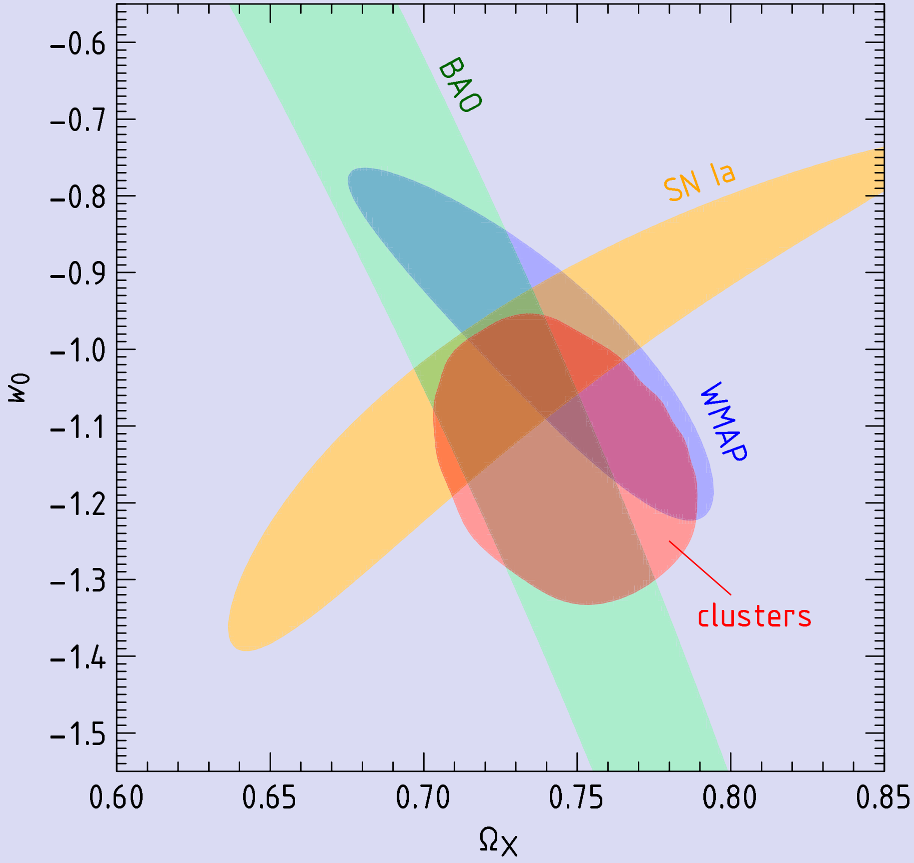
XLF(survey+follow-up data): BCS +REFLEX+MACS (z<0.5) 238 clusters (Mantz et al 10a). Including systematics

Ω_m = 0.23 +- 0.04
σ_8 = 0.82 +- 0.05
w = -1.01 +- 0.20

Both cluster experiments combined

Dark Energy results: flat wCDM

Vikhlinin et al 10



Green: BAO
Blue: CMB (WMAP)
Red: Clusters
Gold: SNIa

Ω_m = 0.26 +- 0.08
σ_8 = 0.81 +- 0.04
w = -1.14 +- 0.21

Beyond ΛCDM: Neutrino properties

Neutrinos and Cosmology

- Neutrino flavor oscillation experiments (solar, atmospheric, reactors) have conclusively shown that the neutrino mass eigenstates are non-degenerate (e.g. Fukuda et al 98, Ahn et al 03, 06, Sanchez et al 03, Aharmim et al 05, Beringer et al 12, etc.). However, measuring the absolute mass scale is still challenging.

- Three 'normal' neutrino species: ν_e, ν_μ, ν_τ. There are though some hints for possible additional, sterile neutrinos from oscillation data (Kopp et al 11, Huber 11, etc.). Recently, CMB observations also seem to favor the presence of additional radiation at the time of decoupling over that from photons and the three 'normal' neutrino species.

- Current constraints from the laboratory experiments: lower bound on $M_\nu=\Sigma_i m_i$ (sum of the masses of the different species) of ~0.056 (0.095)eV/c^2 for the normal (inverted) hierarchy; and an upper bound of ~6eV/c^2 (from hereon c=1). The Heidelberg-Moscow experiment has limited the mass of the electron neutrino to <0.35eV (Klapdor-Kleingrothaus & Krivosheina 06).

Neutrinos and Cosmology

- Neutrinos play an important role in the early universe and therefore affect cosmological observations (review: Lesgourges & Pastor 06).

- The primary cosmological effect of the non-zero neutrino mass is to suppress the formation of cosmic structure on intermediate and small scales. CMB contains information on LSS at early times. The combination with probes today give good constraints on the absolute neutrino mass scale.

- Interference with dark energy and inflation physics. Combining experiments helps.

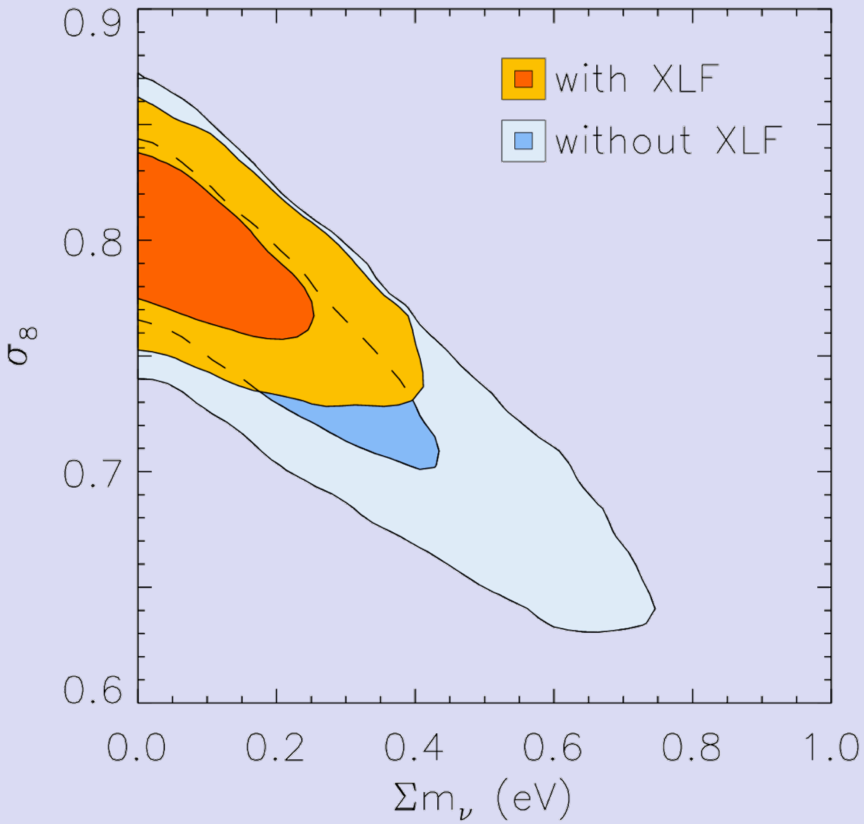
- Combined cosmological observations: $\Sigma_i m_i <\sim 0.3\text{-}0.6\,eV$.

- Neutrino oscillation experiments favor a large mass for sterile neutrinos yielding a lower limit on their mass of 1eV which is incompatible with cosmological observations. This can be alleviated with for example initial lepton asymmetry (Hannestad et al 12).

Robust constraints on neutrino properties

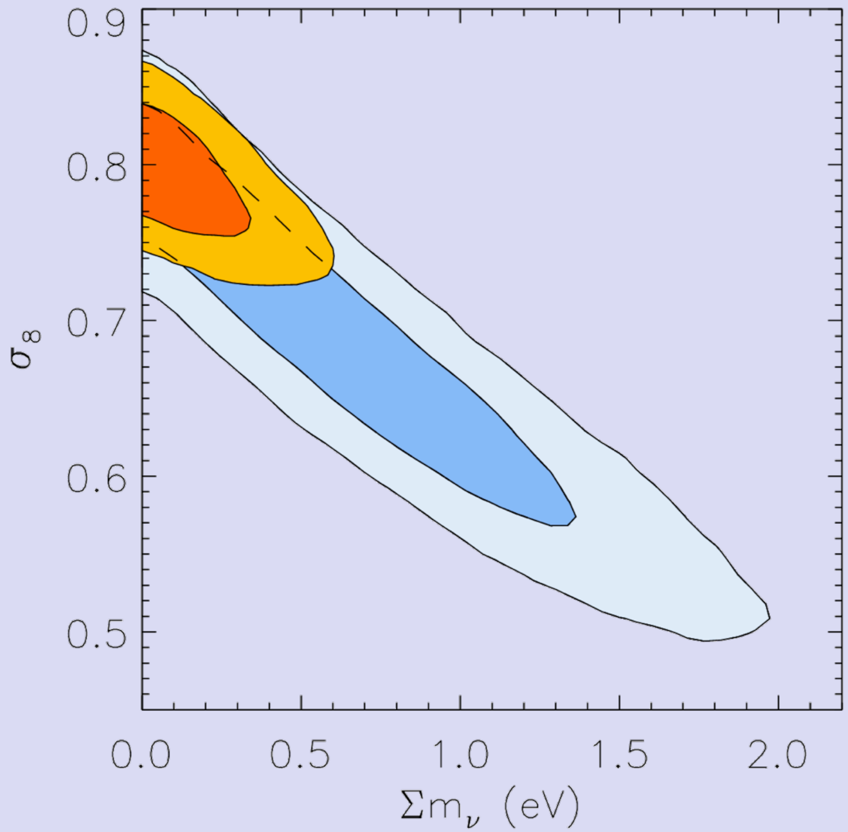
ΛCDM+Σm_ν: Breaking the degeneracy in the Σm_ν, σ_8 plane

Σm_ν<0.33eV (95.4%)



Even more useful when allowing N_{eff}, Ω_k, r, n_t (tensors) to be free

Σm_ν<0.7eV (95.4%) N_{eff}=3.7+-0.7 (68.3%)



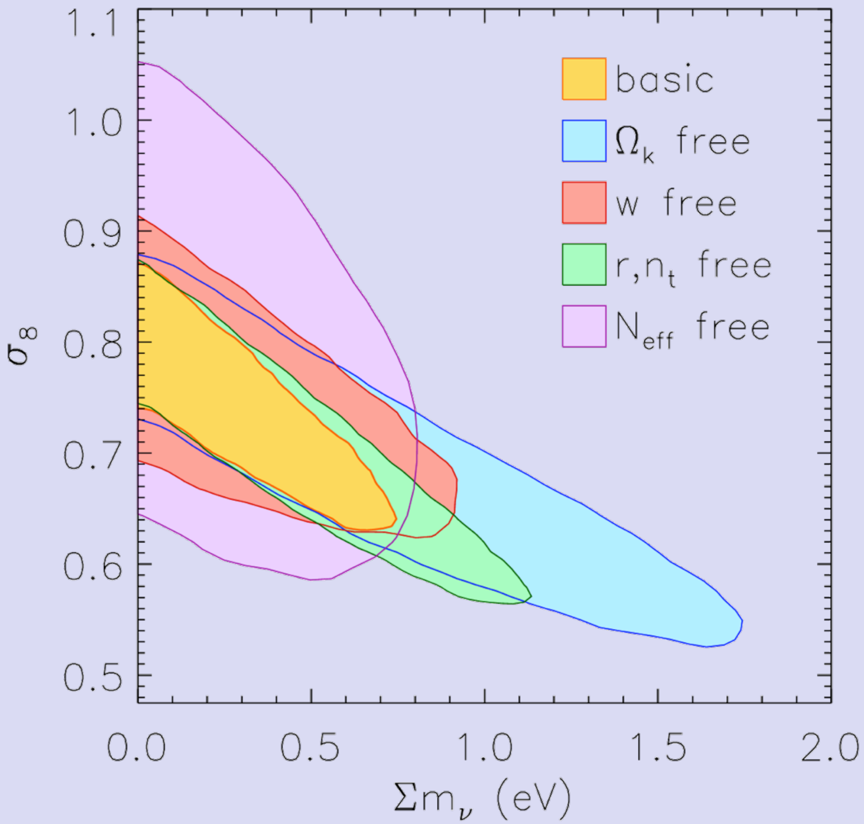
Note differences in scale between panels

Mantz et al 10c

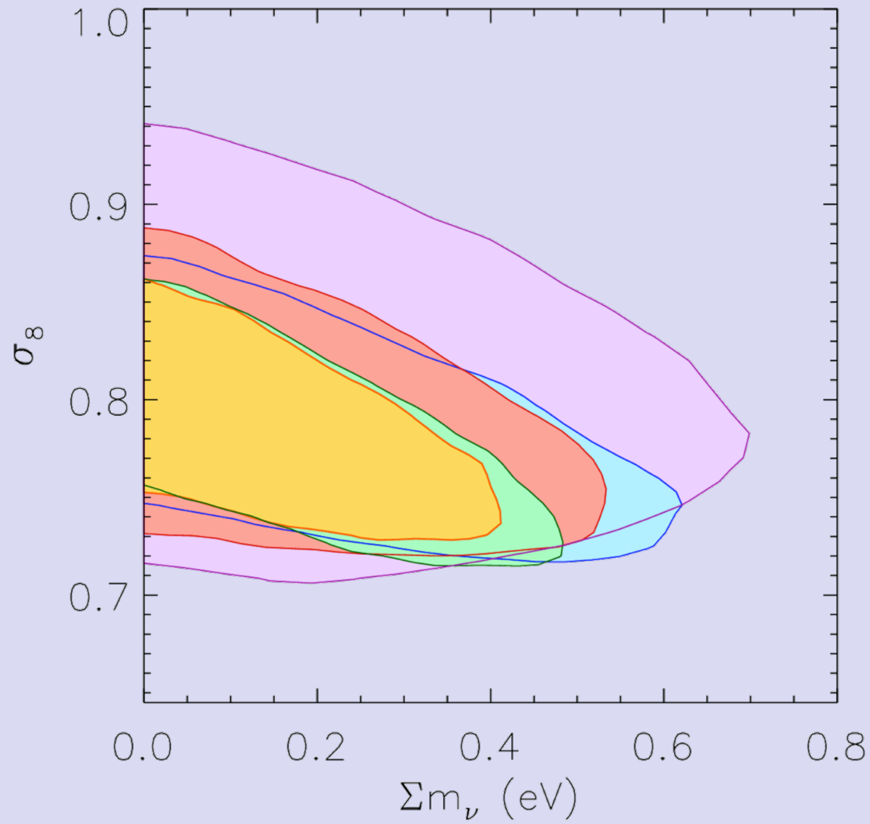
Robust constraints on neutrino properties

Basic: ΛCDM+Σm_ν

CMB+fgas+SNIa+BAO



CMB+fgas+SNIa+BAO+XLF

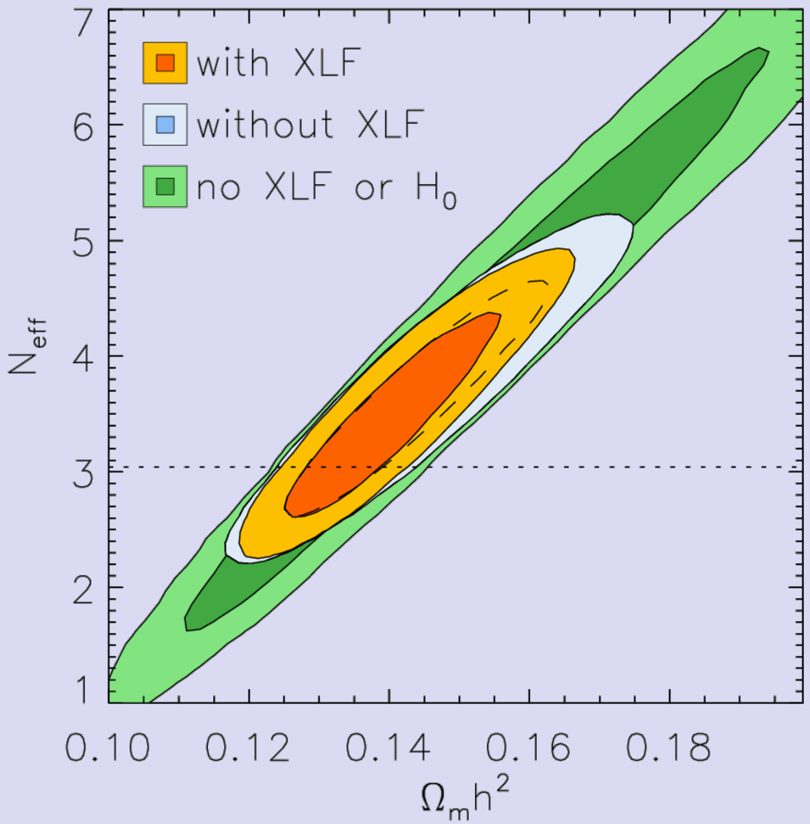


Mantz et al 10c

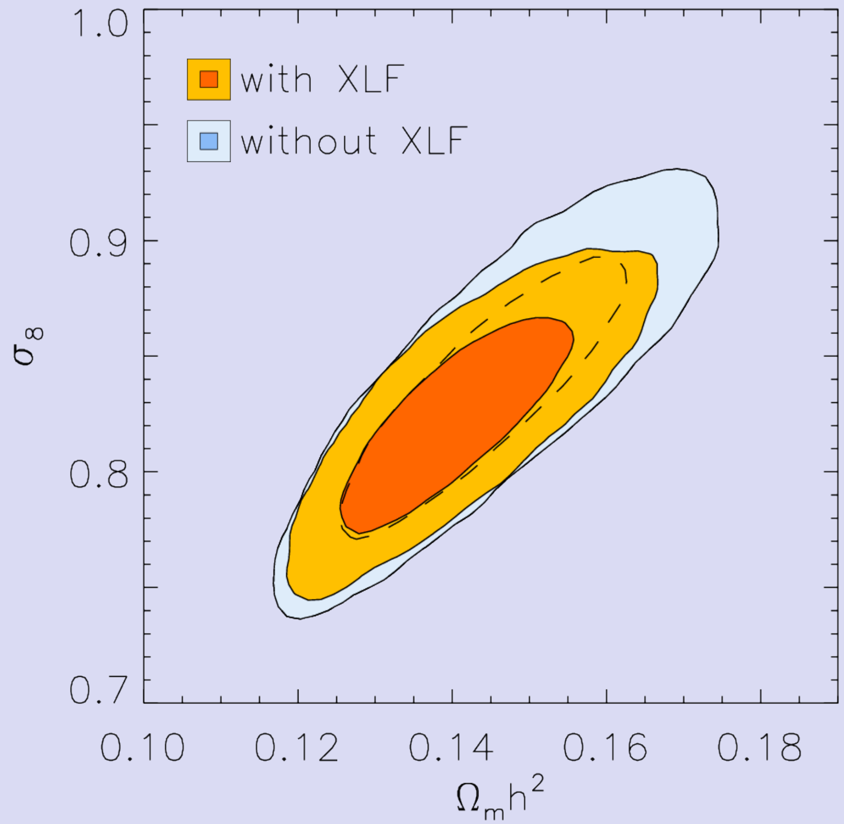
Robust constraints on neutrino properties

CMB+fgas+SNIa+BAO

ΛCDM+N_{eff}



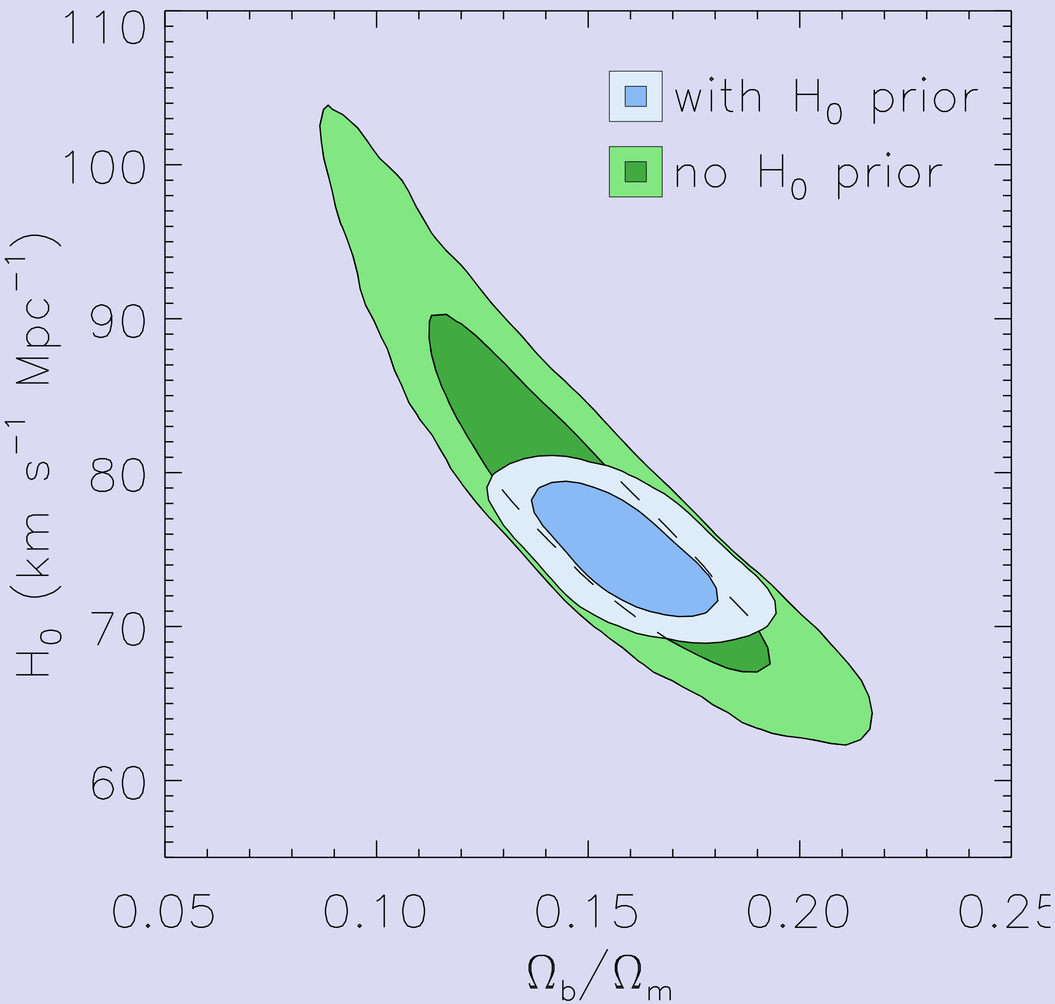
ΛCDM+N_{eff}+M_ν +Ω_k+r+n_t



Mantz et al 10c

Breaking degeneracies with other data sets

Mantz et al 10c

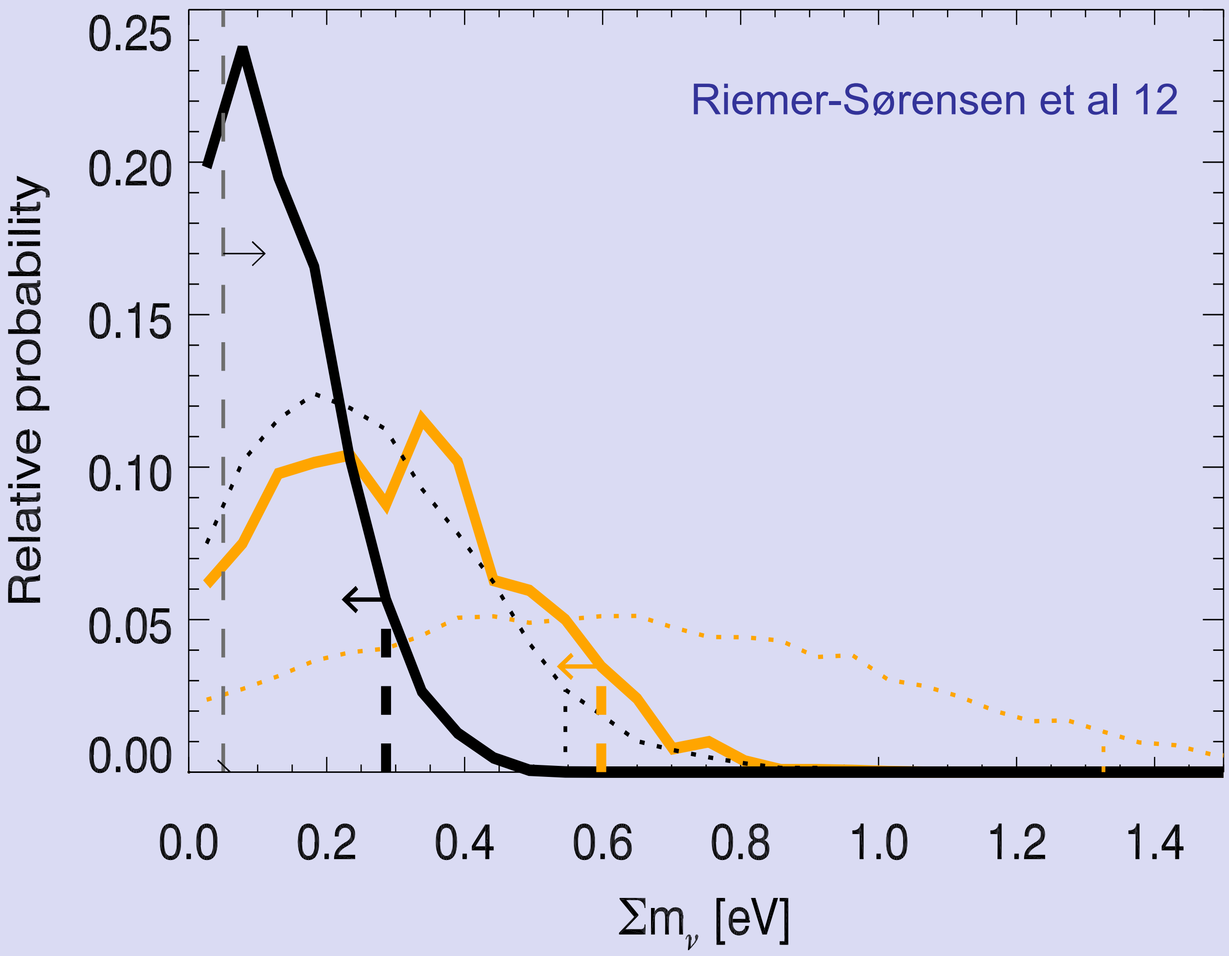


N_{eff} is free

Green contours: CMB+fgas +SNIa+BAO (strong degeneracy).

Blue contours: adding H_0 at the 5% level helps significantly with this degeneracy.

Other cosmological constraints on neutrinos



Dotted orange: WMAP

Solid orange: WiggleZ +WMAP

Dotted black: WMAP+BAO +H_0

Solid black: WiggleZ+WMAP +BAO+H_0

Dashed grey: lower limit, oscillation experiments

Other vertical lines: 95% confidence upper limits

Summary

- For the first time, we present a simultaneous and self-consistent analysis of cluster survey plus follow-up data accounting for survey biases, systematic uncertainties and parameter covariances. This kind of analysis is essential for both cosmological and scaling relation studies.

- We obtain the tightest constraints on w for a single experiment from measurements of the growth of cosmic structure in clusters (flat wCDM): w = -1.01+-0.2.

- We use follow-up Chandra and ROSAT data for a wide redshift range of clusters and gas mass as total mass proxy (f_{gas} has low scatter), which is crucial to obtain such tight constraints. We obtain not only important cosmological but also astrophysical results for clusters.

- Our results highlight the importance of X-ray cluster data to test dark energy and modified gravity models as well as neutrino properties.

- The same techniques developed here can be applied to SZ and optical surveys.

- Future: more MACS and Chandra data, XCS, XXL, Astro-H, eROSITA, Athena, WFXT.

Modern cosmology with X-ray luminous clusters of galaxies

Thursday Lecture: Cosmological Models and Modeling

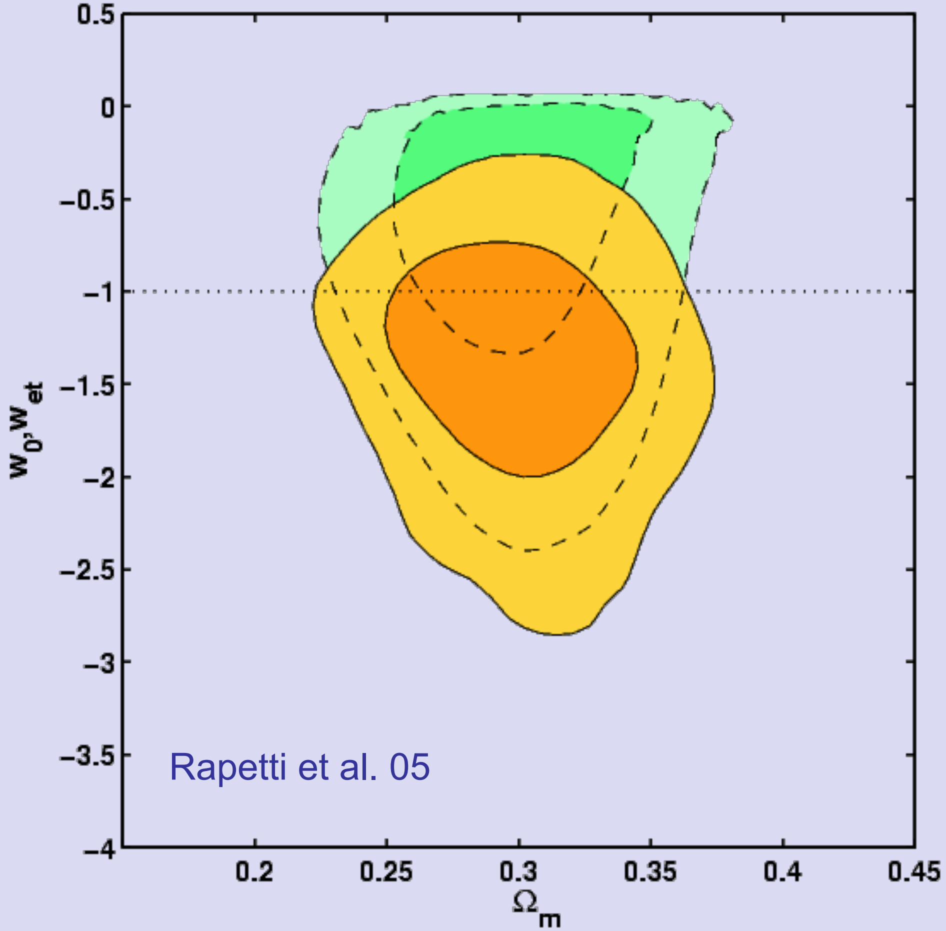
David Rapetti
DARK Fellow
Dark Cosmology Centre, Niels Bohr Institute
University of Copenhagen

Models and probes of cosmic acceleration

- Some recent dark energy reviews:
  - Copeland, Sami, Tsujikawa, 06, Int. J. Mod Phys D
  - Frieman, Turner, Huterer, 08, Ann. Rev. Astr. & Astrophys., 46, 385
  - Weinberg, Mortonson, Eisenstein, Hirata, Riess, Rozo, 12, for Phys. Reports, arXiv:1201.2434
- Dark energy task forces and future dark energy missions:
  - Albrecht, Bernstein, Cahn, Freedman, Hewitt, Hu, Huth, Kamionkowski, Kolb, Knox, Mather, Staggs, Suntzeff, 06, arXiv/0609591
  - Albrecht, Amendola, Bernstein, Clowe, Eisenstein, Guzzo, Hirata, Huterer, Kirshner, Kolb, Nichol, 09, arXiv:0901.0721
  - Amendola, et al (Euclid Satellite), 12, arXiv:1206.1225

Beyond ΛCDM: Evolving dark energy w(z)

Constraints on w_0, w_{et} marginalizing over z_t



Combined constraints (marginalized 68%)

Ω_m = 0.299 + 0.029 - 0.027

w_0 = -1.27 + 0.33 - 0.39

w_{et} = -0.66 + 0.44 - 0.62

WMAP1+CBI+ACBAR

SNIa: Riess et al 04

f_{gas}: Allen et al 04

marginalized over $0.05<z_t<1$

Two parameters:

$w=w_0+w_1(1-a)$ fix transition at $z_t=1$ between w_0 (present) and $w_{et}=w_0+w_1$ (early times).

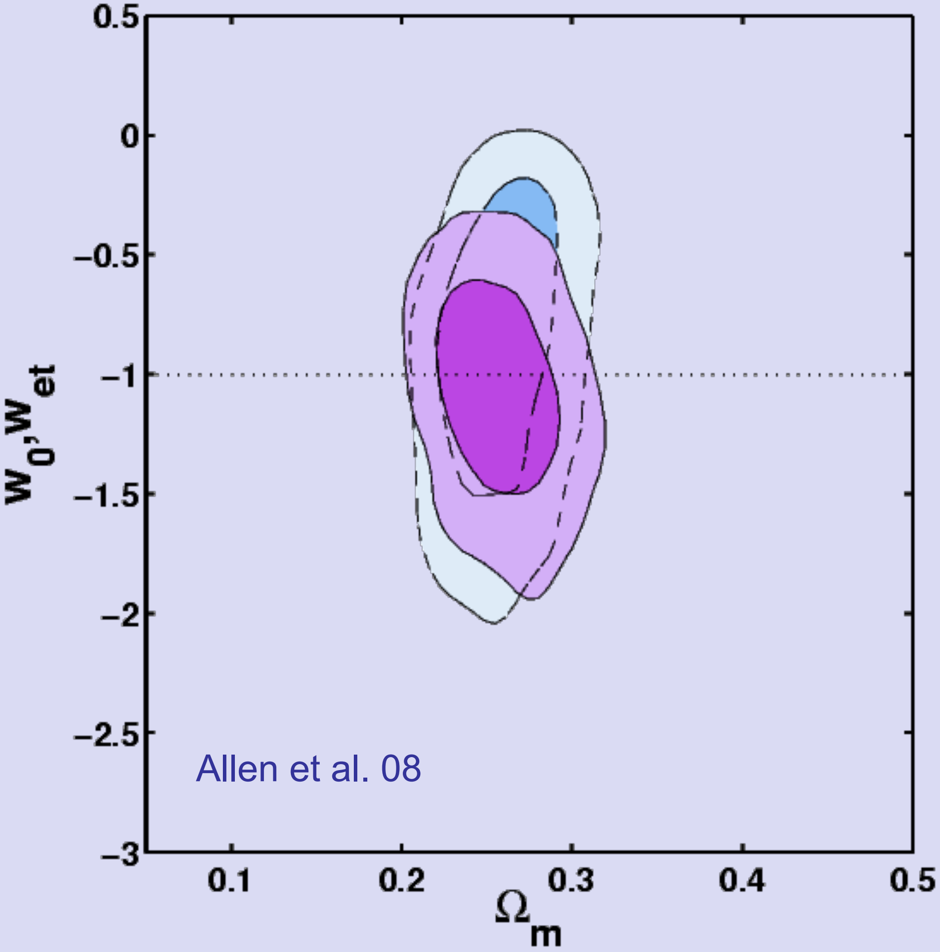
Three parameters:

free transition z_t between w_0 and w_{et}:

$$w = \frac{w_{et} z + w_0 z_t}{z + z_t} = \frac{w_{et}(1-a)a_t + w_0(1-a_t)a}{a(1-2a_t)+a_t}$$

Rapetti et al. 05

Constraints on w_0, w_{et} marginalizing over z_t



Combined constraints (marginalized 68%)

$\Omega_m = 0.254 \pm 0.022$

$w_0 = -1.05 + 0.31 - 0.26$

$w_{et} = -0.83 + 0.48 - 0.43$

WMAP3+CBI+Boomerang+ACBAR

SNIa: Davis et al. 07

f_{gas}: Allen et al. 08

marginalized over $0.05 < z_t < 1$

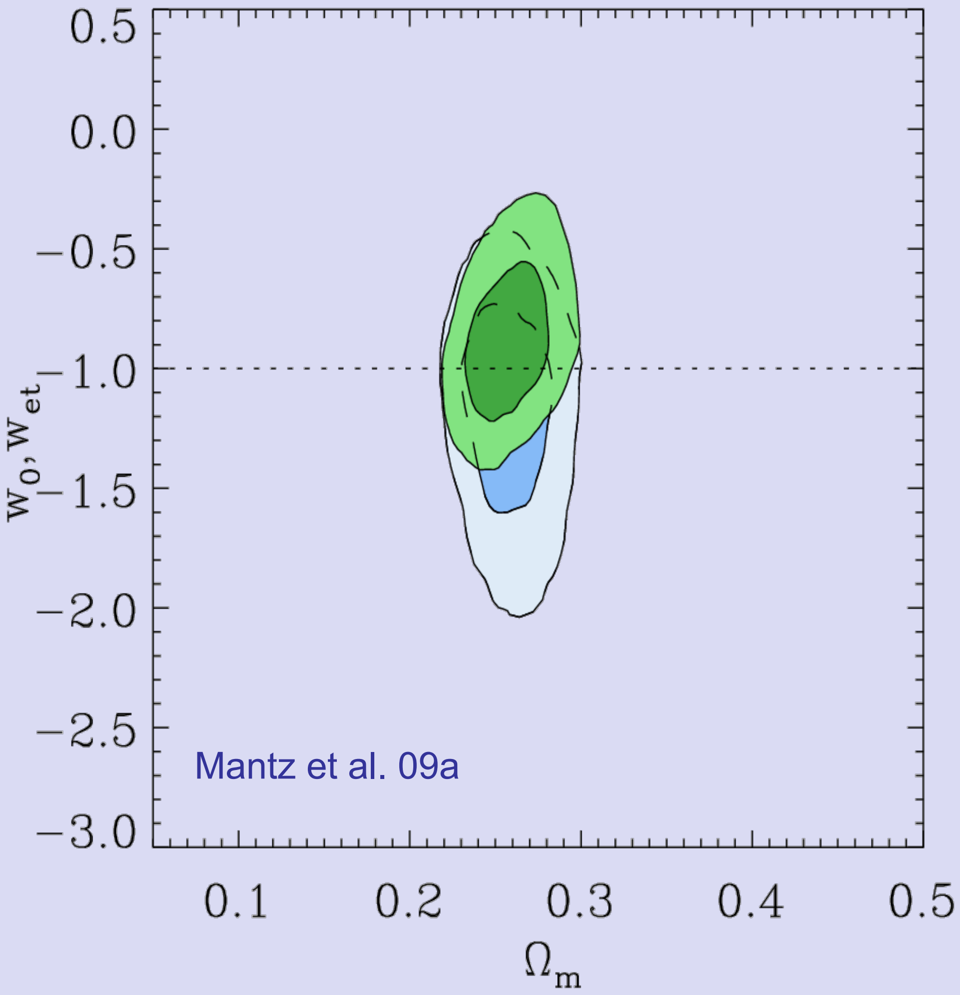
Three parameters:

free transition z_t between w_0 and w_{et}:

$$w = \frac{w_{et} z + w_0 z_t}{z + z_t} = \frac{w_{et}(1-a)a_t + w_0(1-a_t)a}{a(1-2a_t)+a_t}$$

Rapetti et al. 05

Current constraints: evolving w



Combined constraints (marginalized 68%)
Ω_m = 0.257 +- 0.016
w_0 = -0.88 + -0.21
w_{et} = -1.05 + 0.20 - 0.36

WMAP5
SNIa: Kowalski et al. 08
f_{gas}: Allen et al. 08
BAO: Percival et al. 07
XLF: Mantz et al. 09a
marginalized over $0.05<z_t<1$

Three parameters:
free transition z_t between w_0 and w_{et}:

$$w = \frac{w_{et} z + w_0 z_t}{z + z_t} = \frac{w_{et}(1-a)a_t + w_0(1-a_t)a}{a(1-2a_t) + a_t}$$

Rapetti et al. 05

"A kinematical approach to dark energy studies"

MNRAS 375 (2007) 1510-1520,

David Rapetti, Steve Allen, Mustafa Amin, Roger Blandford

Why kinematical approaches?

- Do not assume any particular gravity theory.

  - Most of current cosmological analyses are dynamical, use the Friedmann equations (and General Relativity) employing Ω_m and w as model parameters.

  - Other dynamical approaches use modified gravity theories.

- Describe directly the expansion history of the Universe, a(t).

  - We measure a late-time cosmic acceleration.

  - It is important now to measure kinematically a transiton to a decelerating phase at earlier times.

Using distance measurement to constrain cosmic acceleration

To SNe Ia z<1.7 (Riess et al. 2004), z<1 (Astier et al. 2005)

The apparent magnitude with redshift:

$$\mu^{th}(z) = 5\log_{10} D(z;\theta) + \mu_0$$

$$d_L(z;\theta) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{E(z;\theta)}$$

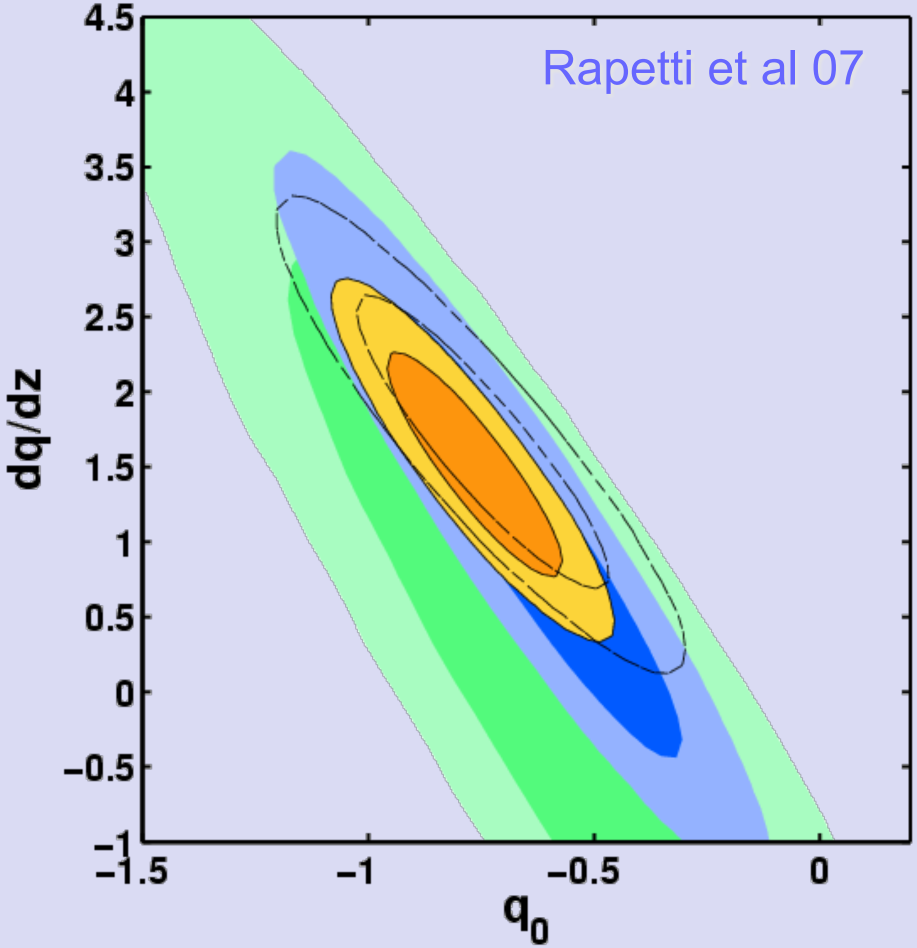
To 41 X-ray clusters 0.06<z<1.07 (Allen et al. 2008)

The apparent evolution of $f_{gas}=M_{gas}/M_{tot}$ with redshift:

$$f_{gas}^{ref}(z) = F\left[\frac{d_A^{ref}(z)}{D_A^{\mathrm{mod}}(z;\theta)}\right]^{1.5}$$

$$F = \frac{b\Omega_b H_0^{1.5}}{\Omega_m (1+0.19\sqrt{h})}$$

Constraints on the deceleration parameter using the three data sets

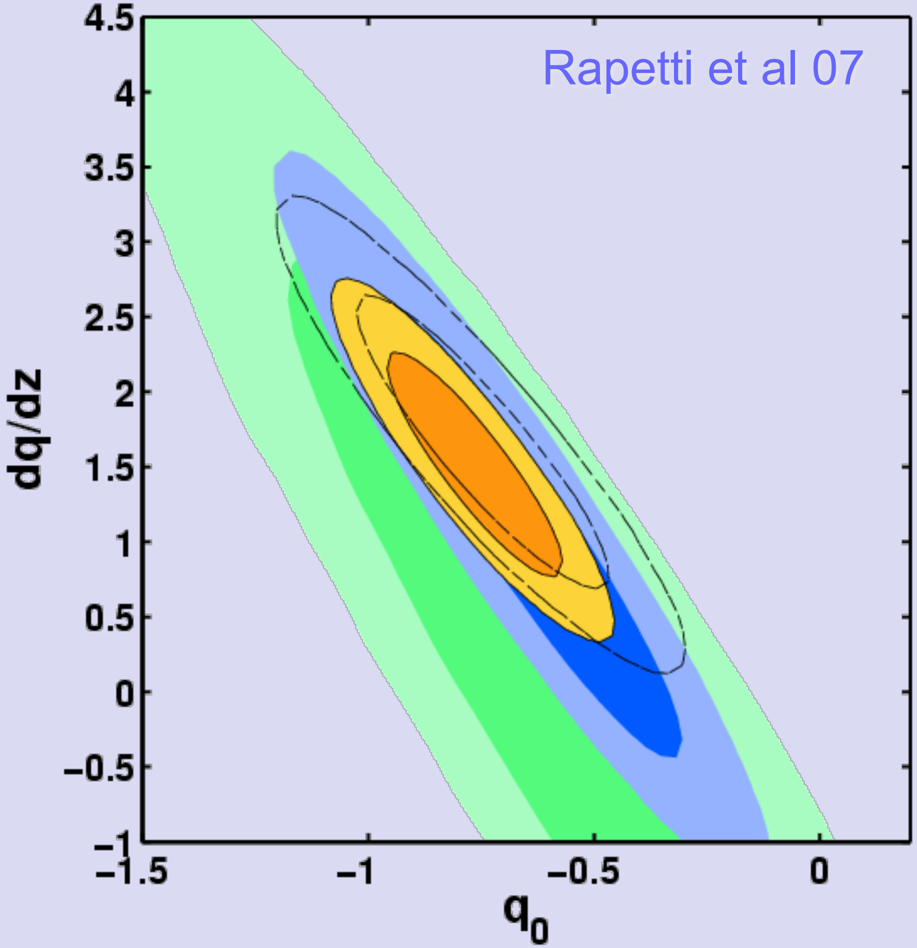


- Using $q(z)=q_0+z(dq/dz)$ as Riess et al 04 we note that the three independent data sets overlap and combined give tight constraints.

- Clusters (green contours) ; SNLS SNIa (blue contours) ; Gold SNIa sample (dashed contours); all combined (orange contours)

- Shapiro & Turner 05 and Elgaroy & Multamaki 06 also used other q(z) parameterizations.

Constraints on the deceleration parameter using the three data sets



•However, the choice of a particular parameterization of q(z) is quite arbitrary.

•And in general does not have a direct meaningful physical interpretation.

Our kinematical formalism: (q_0,j) parameter space

Deceleration parameter

$$q(t) = -\frac{1}{H^2}\left(\frac{\ddot{a}}{a}\right) \qquad q(a) = -\frac{1}{H}(aH)'$$

Jerk parameter

$$j(t) = \frac{1}{H^3}\left(\frac{\dddot{a}}{a}\right) \qquad j(a) = \frac{\left(a^2H^2\right)''}{2H^2}$$

$$a^2V''(a) - 2j(a)V(a) = 0 \leftarrow V(a) \equiv -\frac{a^2H^2}{2H_0^2}$$

$$V(1) = -\frac{1}{2} \qquad V'(1) = -\frac{H_0'}{H_0} - 1 = q_0$$

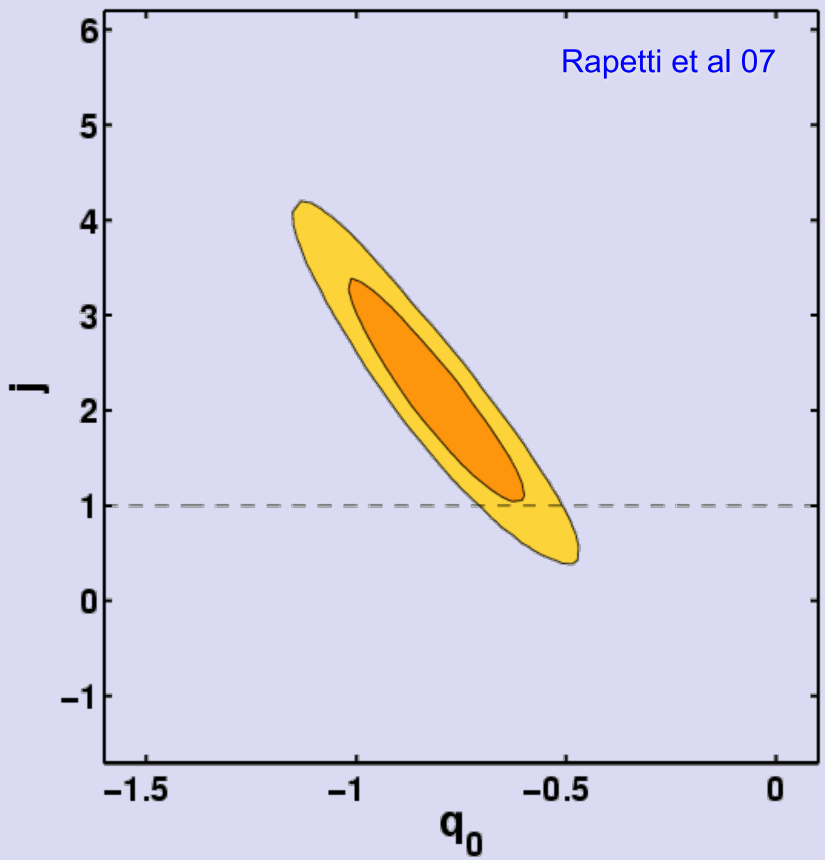
j(a)=1 corresponds to all ΛCDM models

For example, for constant j models we get

$$V(a) = -\frac{\sqrt{a}}{2}\left[\left(\frac{p-u}{2p}\right)a^p + \left(\frac{p+u}{2p}\right)a^{-p}\right] \qquad p \equiv \frac{1}{2}\sqrt{1+8j} \qquad u \equiv 2(q_0 + 1/4)$$

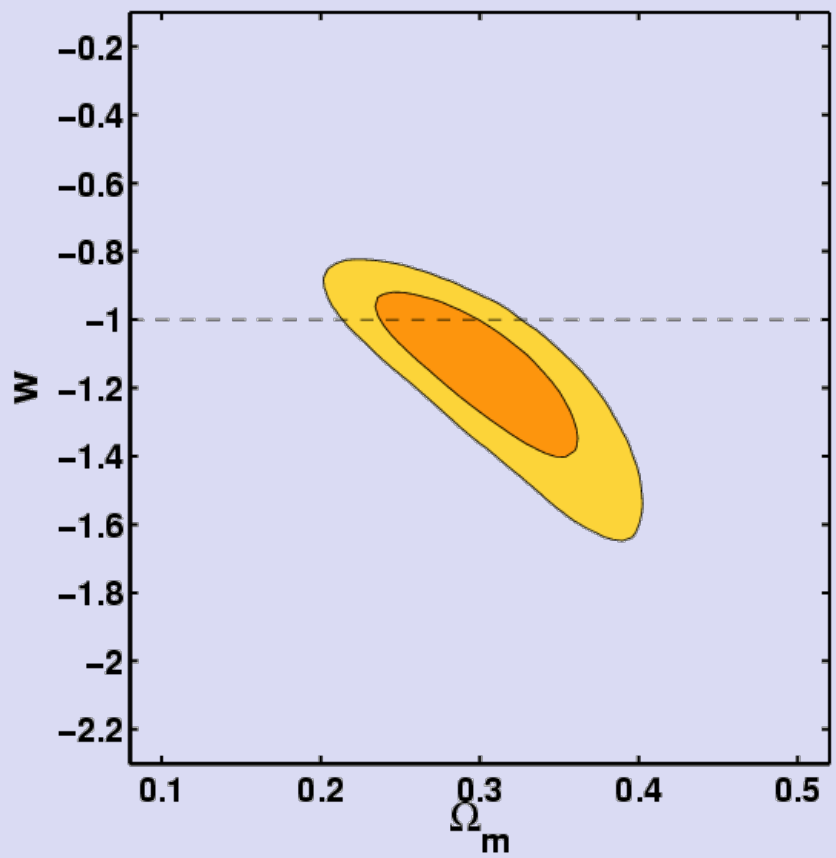
Basic kinematical and dynamical models combining all three data sets

Constant j model



q_0 = -0.81 +- 0.14
j = 2.16 +0.81- 0.75

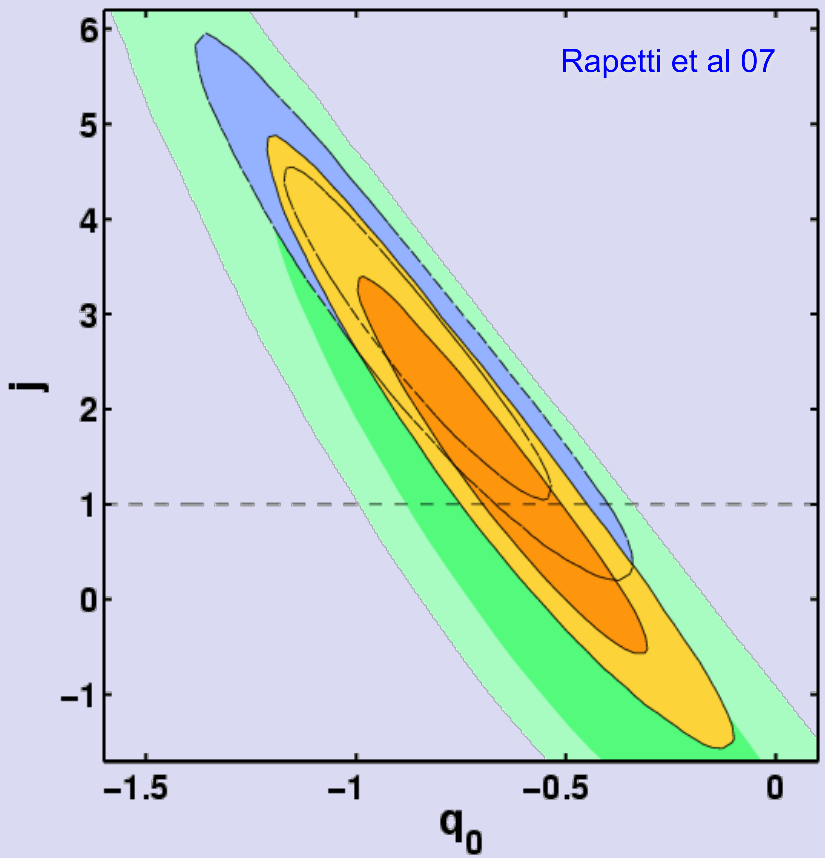
Constant w model



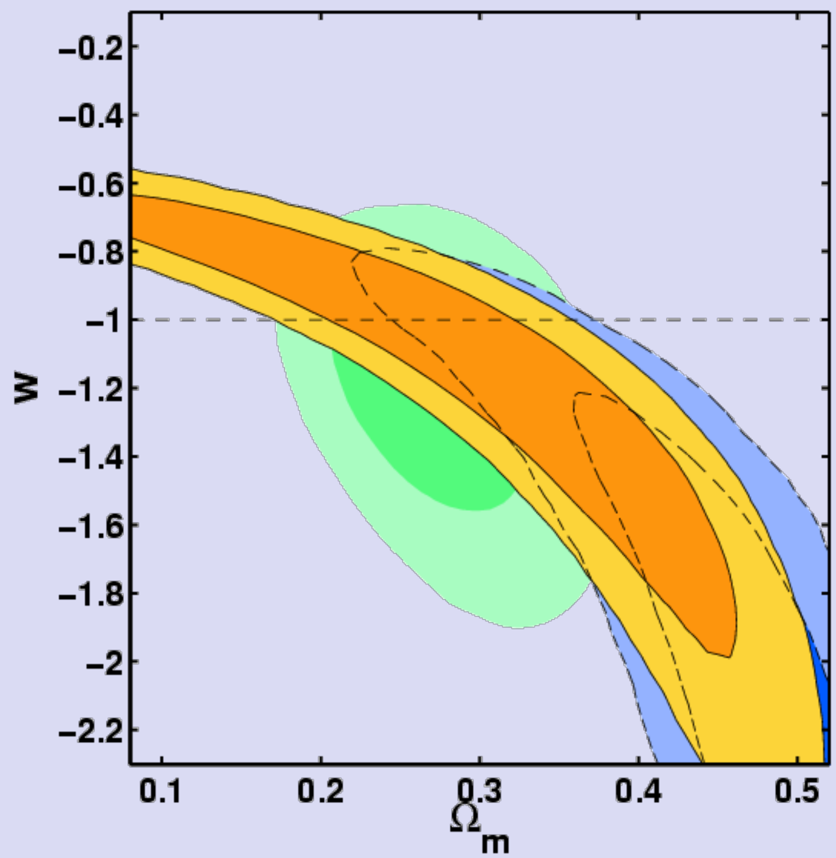
Ω_m = 0.306 +0.042- 0.040
w = -1.15 +0.14- 0.18

Basic kinematical and dynamical models for each data set

Constant j model

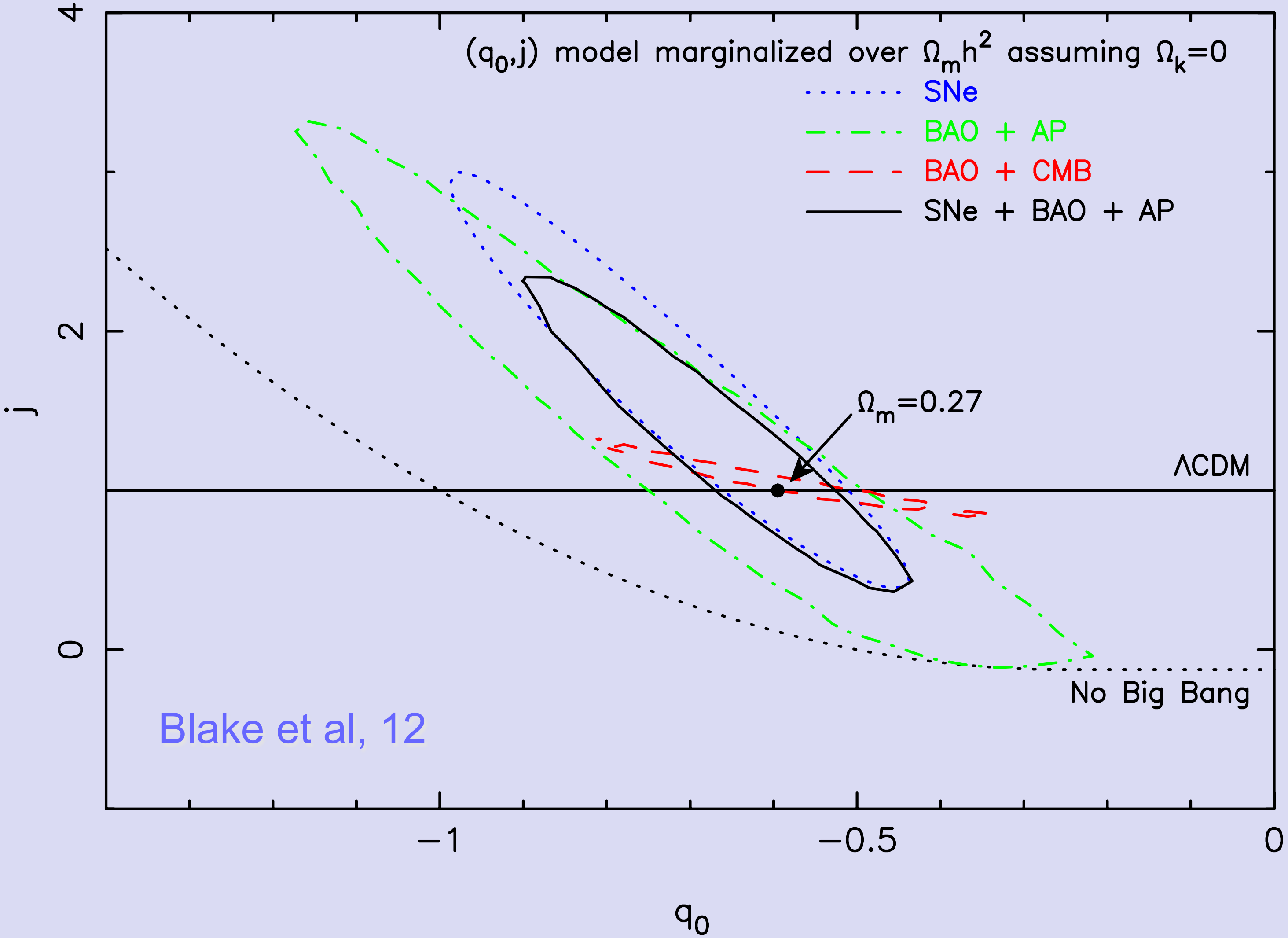


Constant w model

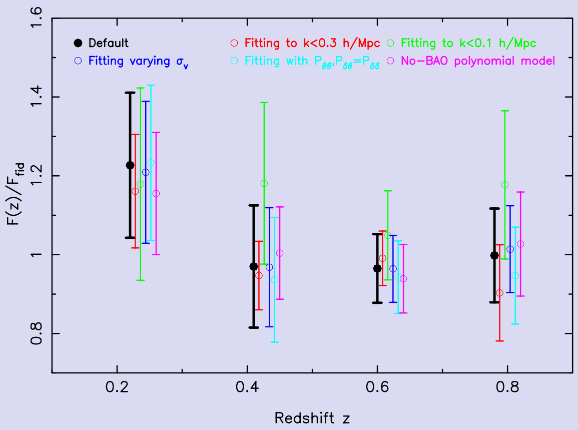


Gold, blue : SNLS, orange : Clusters, green

Recent results on the kinematical model for various combinations of data sets

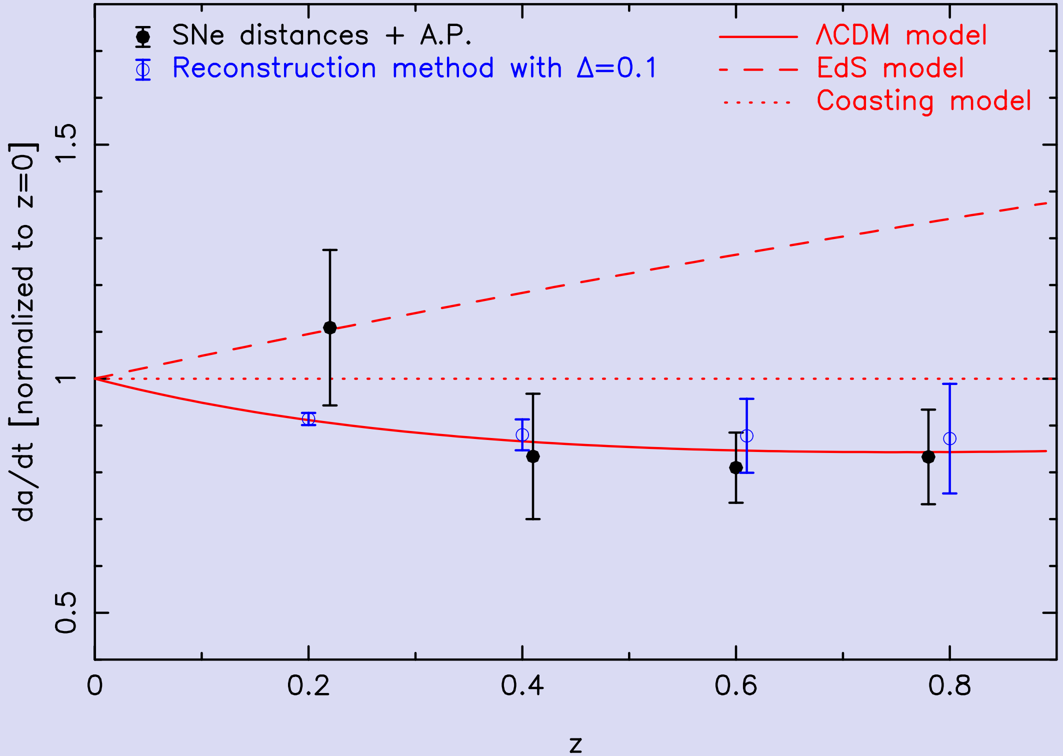


Alcock-Paczynski test data from WiggleZ data



$$F(z) = (1+z)D_A(z)H(z)/c$$

$D_A(z)$: angular diameter distance and
$H(z)=H_0E(z)$: Hubble parameter

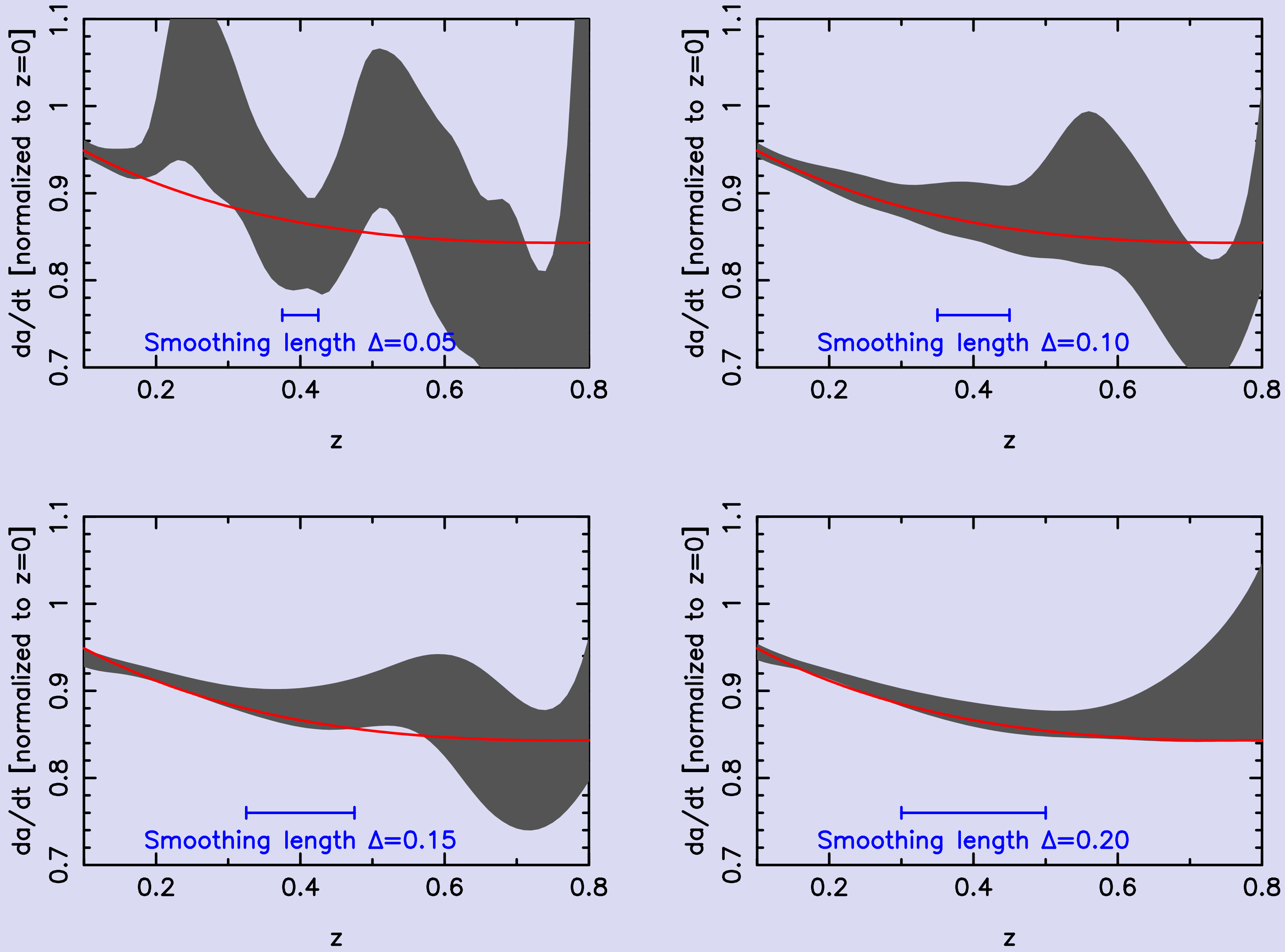


Blake et al, 11

SNe+AP effect

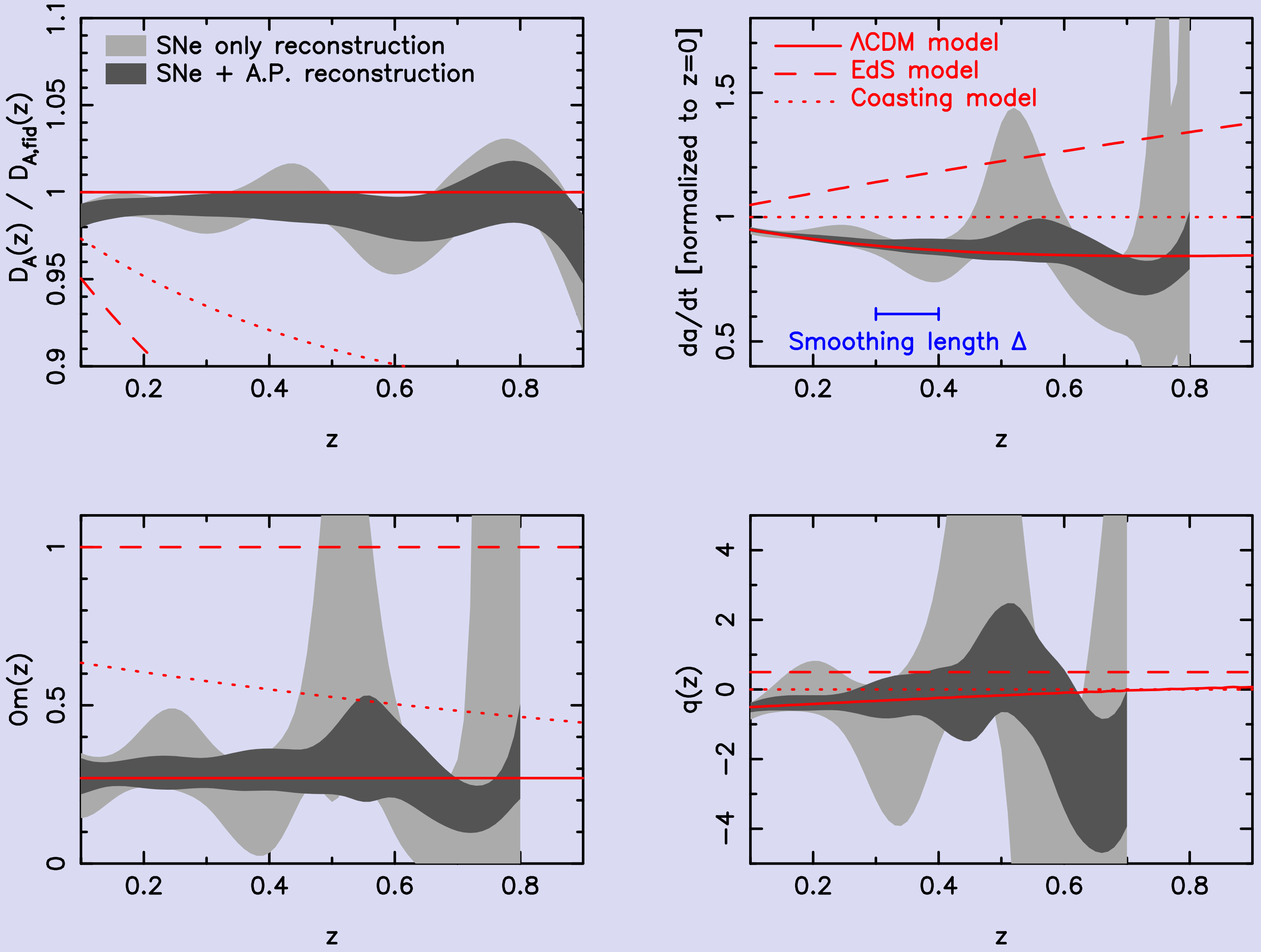
Reconstruction of kinematical quantities

Blake et al, 11



Non-parametric reconstruction of the cosmic expansion history

Blake et al, 11



In an analogous manner to dynamical studies which allow w(a) we search for evolution in j(a) as a model parameter.

$$j(a;C) = j^{\Lambda CDM} + \Delta j(a;C) \qquad j^{\Lambda CDM} = 1$$

General scheme:

Expanding Δj in Chebyshev polynomials

$$\Delta j(a;C) \approx \sum_{n=0}^{N} c_n T_n(a_c) \qquad [a_{\min} = 0.36, a_{\max} = 1]$$

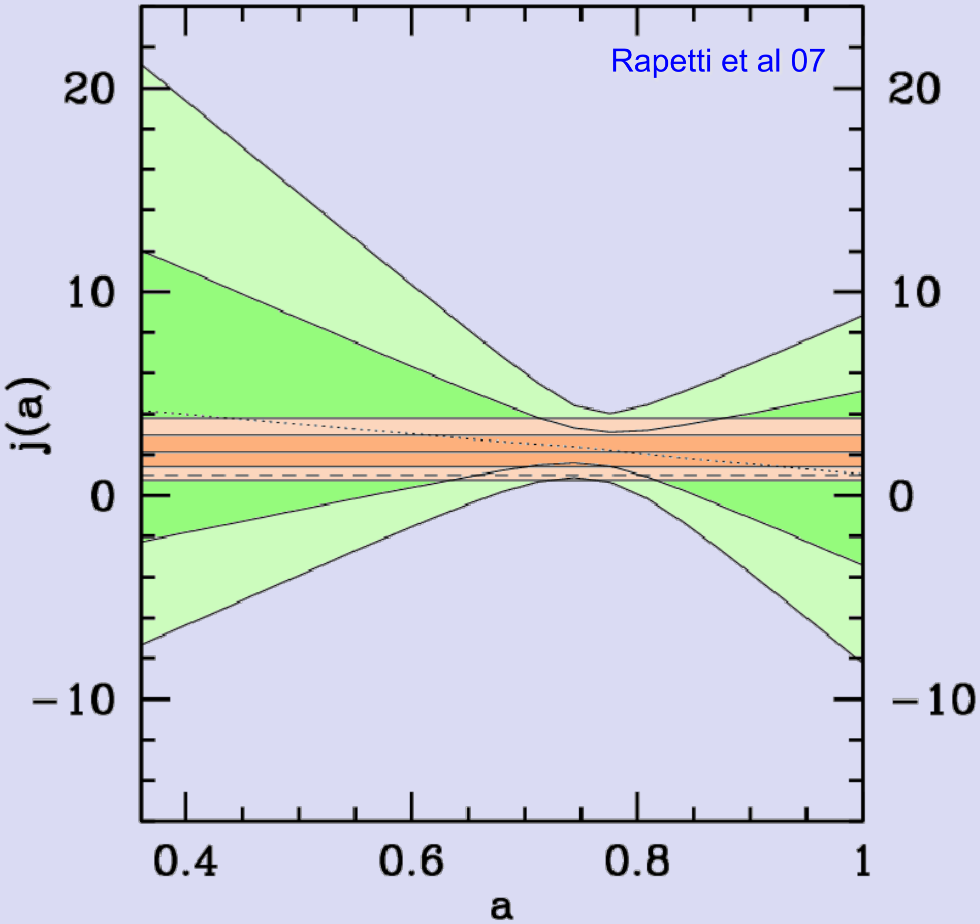
$$a_c \equiv \frac{a - (1/2)(a_{\min} + a_{\max})}{(1/2)(a_{\max} - a_{\min})}$$

$$T_{n+1}(a_c) = 2a_c T_n(a_c) - T_{n-1}(a_c)$$

$$C = (c_0, c_1, c_2, \ldots, c_N)$$

For example $T_0(a_c) = 1, \quad T_1(a_c) = a_c, \quad T_2(a_c) = 2a_c^2 - 1, \quad \ldots$

Constraints on an evolving jerk model



- 68.3 and 95.4 per cent confidence variations about the median values for j(a) over the range [0.36,1].
- Constant jerk model (red contours), [q_0,j(a;c_0,c_1)] model (green contours). The evolving model is not required for current data.
- Dashed line, j(a)=1, i.e. cosmological constant.

Hypothesis testing: How many model kinematical parameters are required?

F-test

$$F = \frac{\Delta\chi^2}{\chi_\nu^2 \Delta m}$$

Bayesian Information Criterion

$$BIC = -2\ln L + k\ln N$$

$$2 < \Delta BIC < 6$$

Bayesian Evidence

$$E(M) \equiv P(D\,|\,M) = \int d\theta P(D\,|\,\theta, M)P(\theta\,|\,M)$$

$$E(M) \approx \frac{1}{N\Delta\theta}\sum_{n=1}^{N} P(D\,|\,\theta_n)$$

$$2.5 < \ln B_{01} < 5$$

Gold+SNLS+Clusters [q0] model -> [q0,j] model

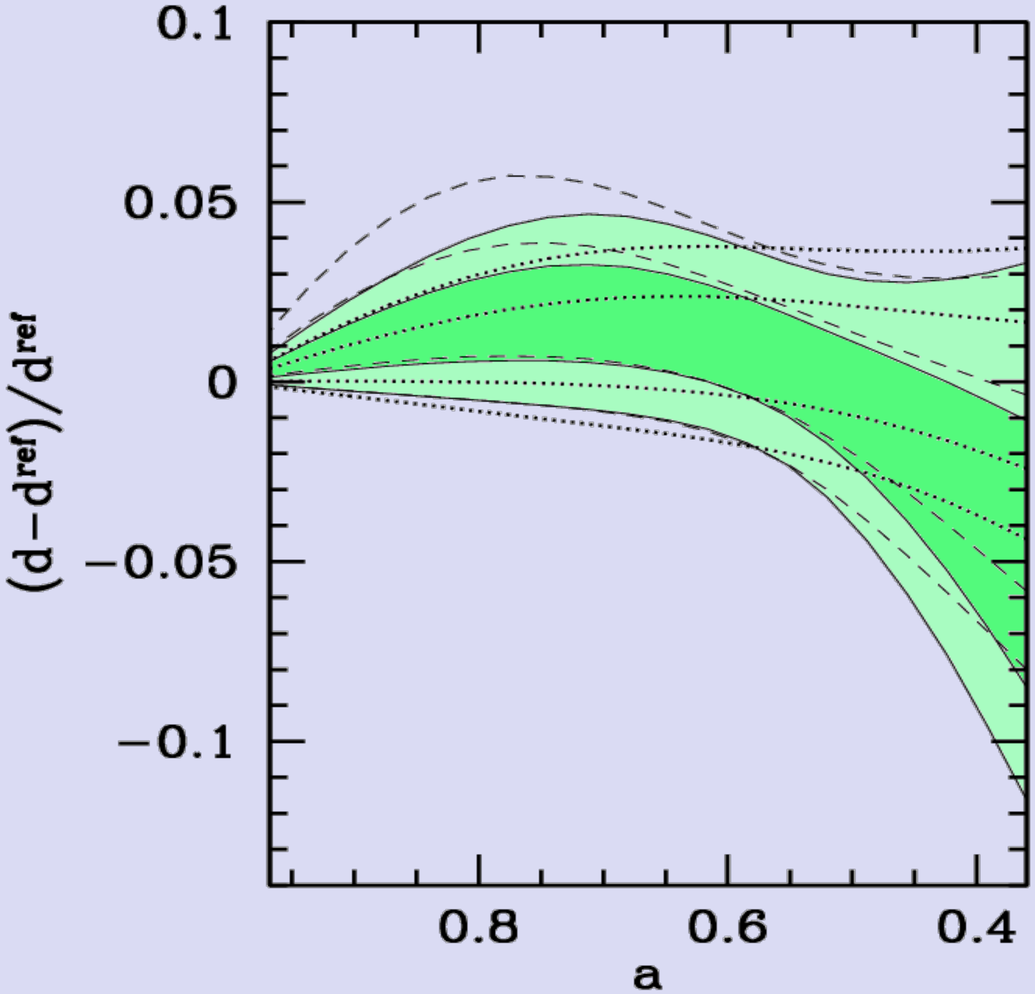
$$\Delta\chi^2 = 10.8$$

$$F - test \rightarrow 99.8\%$$

$$\Delta BIC = 5.1$$

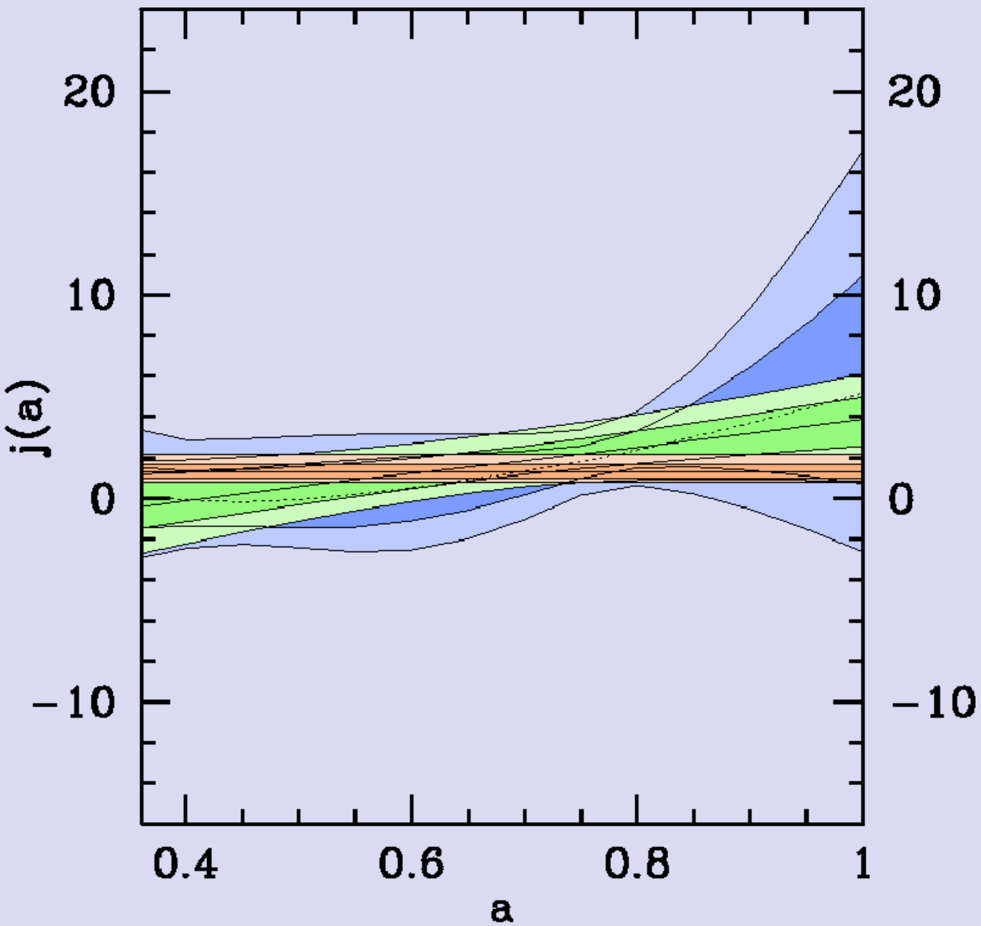
$$\ln B_{JQ} = 3.0$$

Constraints on the distances



- The 68.3 and 95.4 % confidence limits on the offset in distance as a function of a(t) relative to a reference ΛCDM.
- Kinematical, constant j model (green contours);
- Dynamical, constant w model (dotted lines); and relaxing the priors (dashed lines)

Using the distance to the last scattering surface



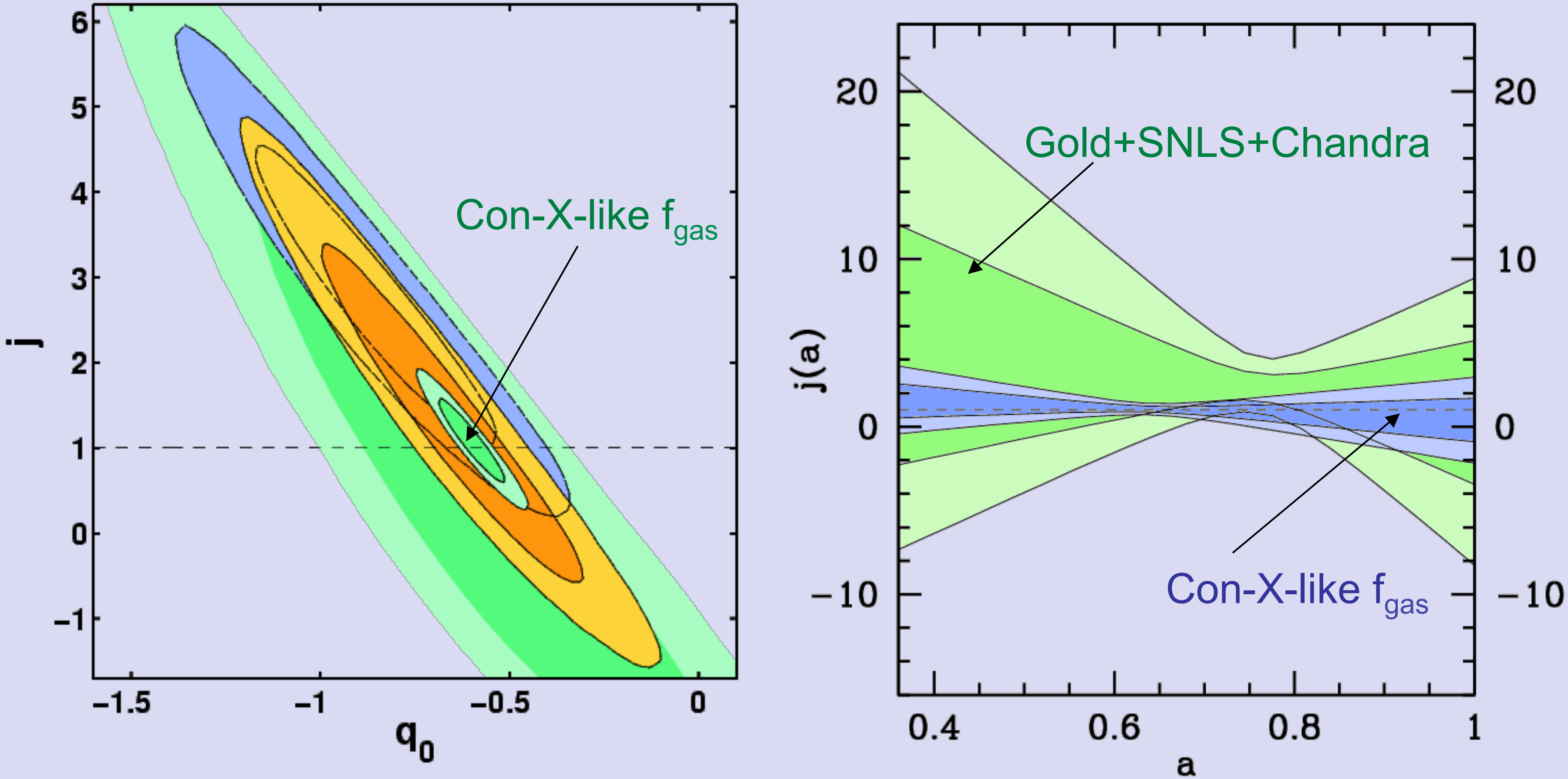
- We use a pseudo-distance measurement to the last scattering surface, $d_A = r_s(a_{dec})/\theta_A$, for illustration purposes.

- Extra strong, though well-motivated, assumptions: dark matter behaves like standard cold dark matter at all redshifts; pre-recombination physics are well described by the standard model; any early dark energy is negligible.

- $r_s(z_{dec})$=146+-10Mpc, comoving $d(z_{dec})$=13.8+-1.1Gpc, z_{dec}=1088. Blue contours for $[q_0, j(a;c_0,c_1,c_2)]$ model.

Projected Con-X-like constraints on:

Kinematics, constant and linear evolving j



Gold sample, blue : SNLS, orange : Clusters, green

$\sigma(q_0)=0.06$ $\sigma(j)=0.33$

Kinematical model

• We have developed a new, natural kinematical parameter space, (q_0,j) to study the expansion history of the Universe.

• We use two independent sets of distance measurements (SNIa and Clusters) making our results more robust against systematics in each individual data set.

• Both models contain a simple representation of ΛCDM ($w=-1$, $j=1$) and both are consistent with it at the 1σ level. This represents an additional support for the ΛCDM paradigm.

• The kinematical framework do not assume any particular gravity theory. The combination of both dynamical and kinematical frameworks may be helpful for distinguishing between dark energy and modified gravity models.

Beyond ΛCDM: Gravity at large scales

"The Observed Growth of Massive Galaxy Clusters III: Testing General Relativity at Cosmological Scales",

MNRAS 406, 1796, 2010

David Rapetti, Steven Allen, Adam Mantz, Harald Ebeling

(Chandra/NASA press release together with Schmidt, Vikhlinin & Hu 09, April 14 2010, "Einstein's Theory Fights off Challengers")

Testing GR on cosmic scales

1. From the evolution of the cluster abundance (XLF) we directly measure linear cosmic expansion and growth.

2. From a variety of measurements we find cosmic acceleration and face the cosmological constant problems.

3. We can either include a new energy component, dark energy, or modify the theory of gravity.

4. We test General Relativity (GR) for consistency.

5. GR has been very well tested from small to Solar system scales. Here we test modifications of GR at cosmological scales.

Ingredients to test a given theory of gravity with cluster abundance data

1. Cosmic expansion model / mean matter density (theory).

2. Matter power spectrum / linear density perturbations (theory).

3. Halo mass function / nonlinear structure formation (N-body simulations for f(R) or DGP: e.g. Schmidt et al 2009, Schmidt 2009a/b, Chan & Scoccimarro 2009, Zhao, Li & Koyama 2011).

4. Relation between the observed mass (e.g. "dynamical") and the true mass (e.g. "lensing") (Theory/N-body simulations: Schmidt 2010a).

Consistency test of the growth rate of General Relativity

1. We use a phenomenological time-dependent parameterization of the growth rate and of the expansion history.

2. We assume the same scale-dependence as GR.

3. We test only for linear effects (not for non-linear effects). We use the "universal" dark matter halo mass function (Tinker et al 2008). Note that the relevant scales for the cluster abundance experiment are at the low end of the linear regime.

4. We match GR at early times and small scales.

Modeling linear, time-dependent departures from GR

$$n(M,z) = \int_0^M f(\sigma)\,\frac{\bar{\rho}_{\rm m}}{M'}\,\frac{d\ln\sigma^{-1}}{dM'}\,dM'$$

Number density of galaxy clusters

$$\sigma^2(M,z) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k,z)|W_M(k)|^2 dk$$

Variance of the density fluctuations

$$P(k,z) \propto k^{n_{\rm s}} T^2(k,z_{\rm t})\,D(z)^2$$

Linear power spectrum

General Relativity

Phenomenological parameterization

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4G\pi\bar{\rho_{\rm m}}\delta$$

$$\frac{d\delta}{da} = \frac{\delta}{a}\Omega_{\rm m}(a)^\gamma$$

GR γ~0.55

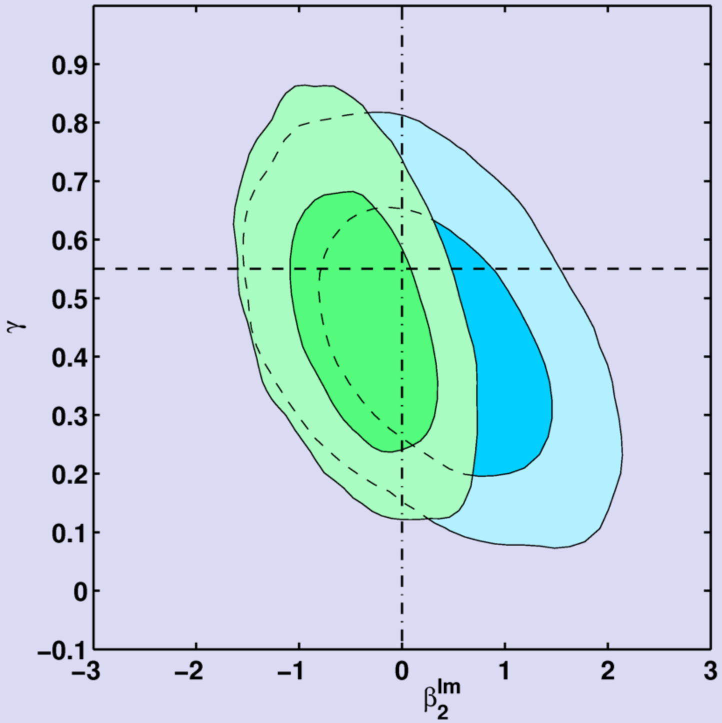
Scale independent in the synchronous gauge

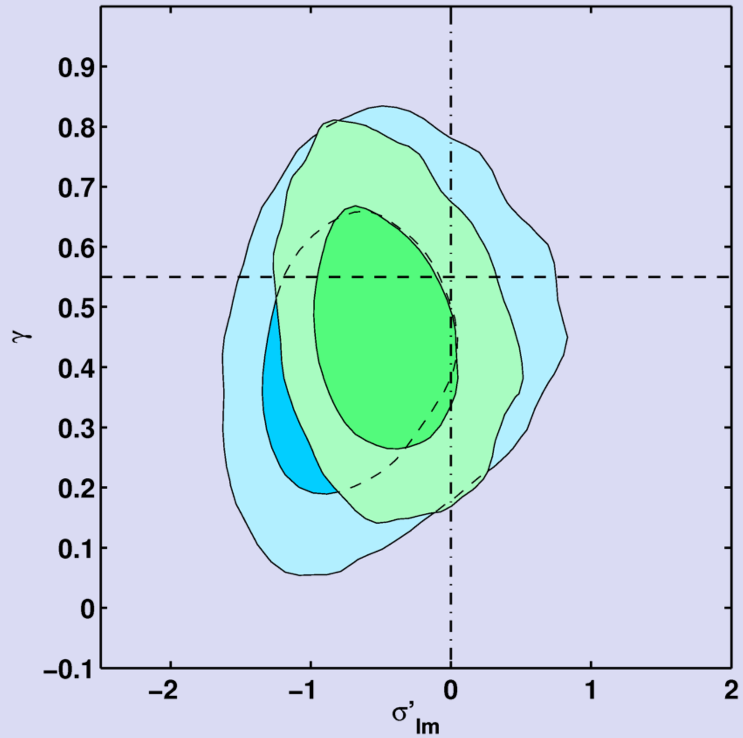
$$f(a) \equiv d\ln\delta/d\ln a = \Omega_{\rm m}(a)^\gamma$$

Growth rate

Test of GR robust w.r.t evolution in the l-m relation

Rapetti et al 10



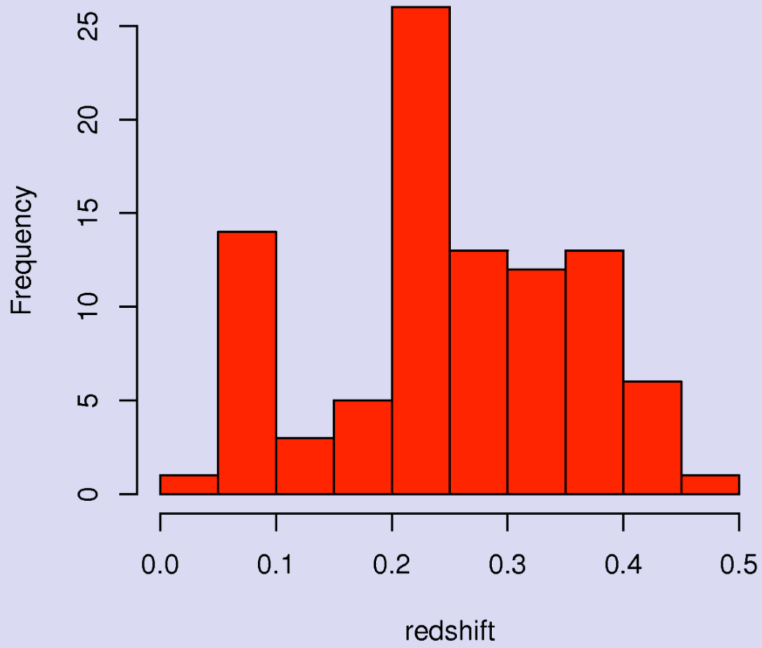


$$\langle l(m)\rangle = \beta_0^{lm} + \beta_1^{lm} m + \beta_2^{lm}\log_{10}(1+z)$$

$$\sigma_{lm}(z) = \sigma_{lm}(1 + \sigma'_{lm} z)$$

Current data do not require (i.e. acceptable fit) additional evolution beyond self-similar and constant scatter nor asymmetric scatter (Mantz et al 2010b).

Investigating luminosity-mass evolution



Within the 238 flux-selected clusters we used pointed observations for

23 clusters (z<0.2) from ROSAT
71 clusters (z>0.2) from Chandra

Mass-luminosity and its intrinsic scatter

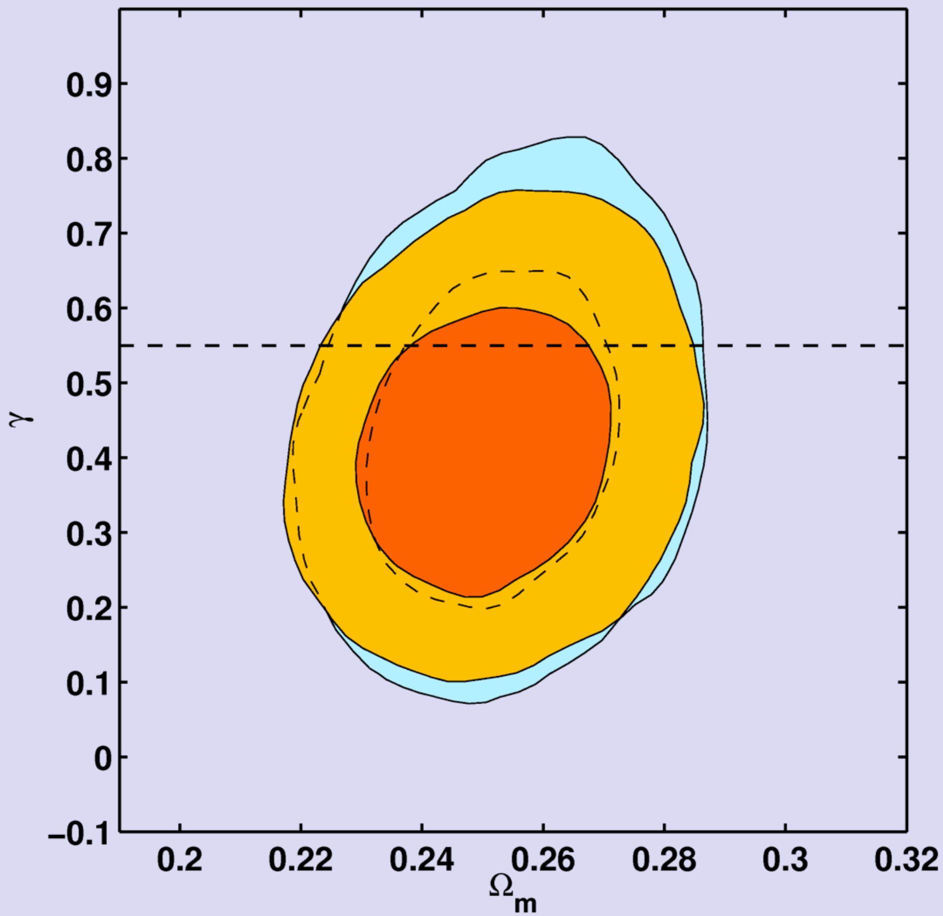
$$\langle l(m) \rangle = \beta_0^{lm} + \beta_1^{lm} m + \beta_2^{lm} \log_{10}(1+z)$$

$$\sigma_{lm}(z) = \sigma_{lm}(1 + \sigma'_{lm} z)$$

$$l = \log_{10}\left(\frac{L_{500}}{E(z) 10^{44} erg\, s^{-1}}\right); \quad m = \log_{10}\left(\frac{M_{500} E(z)}{10^{15} M_{solar}}\right)$$

flat ΛCDM + growth index γ

Rapetti et al 10



XLF: BCS+REFLEX+MACS (z<0.5)
238 survey with 94 X-ray follow-up

CMB (WMAP5)
SNIa (Kowalski et al 2008, UNION)
cluster f_{gas} (Allen et al 2008)

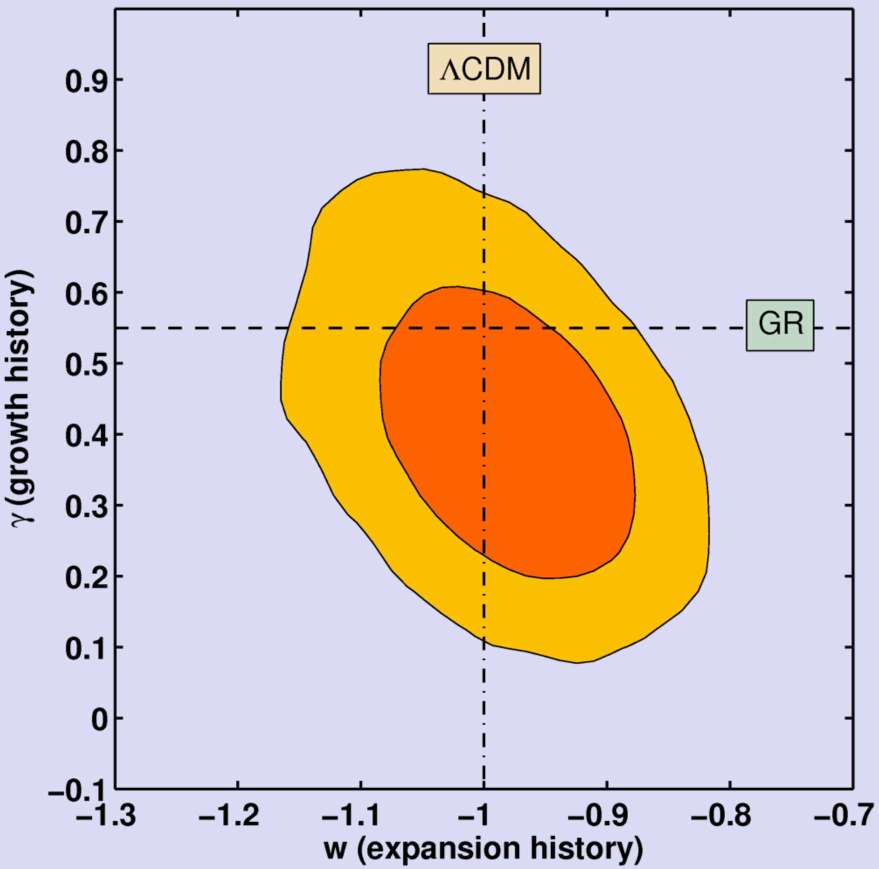
For General Relativity γ~0.55

Gold: Self-similar evolution and constant scatter
Blue: Marginalizing over β^{lm}_2 and σ'_{lm} (only ~20 weaker: robust result on γ).

Remarkably these constraints are only a factor of ~3 weaker than those forecasted for JDEM/ WFIRST-type experiments (e.g. Thomas et al 2008, Linder 2009).

flat wCDM + growth index γ

Rapetti et al 10



XLF: BCS+REFLEX+MACS (z<0.5)

238 survey with 94 X-ray follow-up

CMB (WMAP5)

SNIa (Kowalski et al 2008, UNION)

cluster f_{gas} (Allen et al 2008)

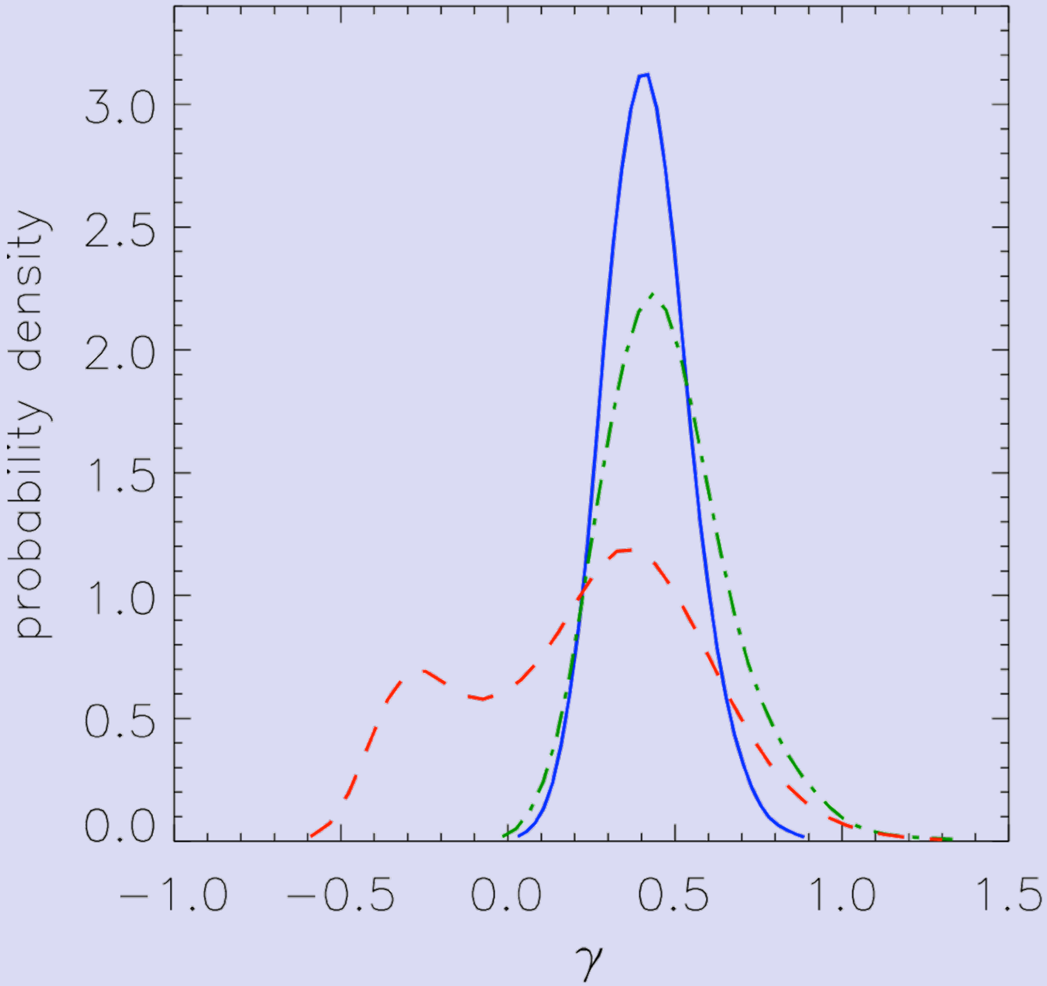
For General Relativity $\gamma \sim 0.55$

Gold: Self-similar evolution and constant scatter

Simultaneous constraints on the expansion and growth histories of the Universe at late times: Consistent with GR+ΛCDM

The impacts of the different data sets

Rapetti et al 10



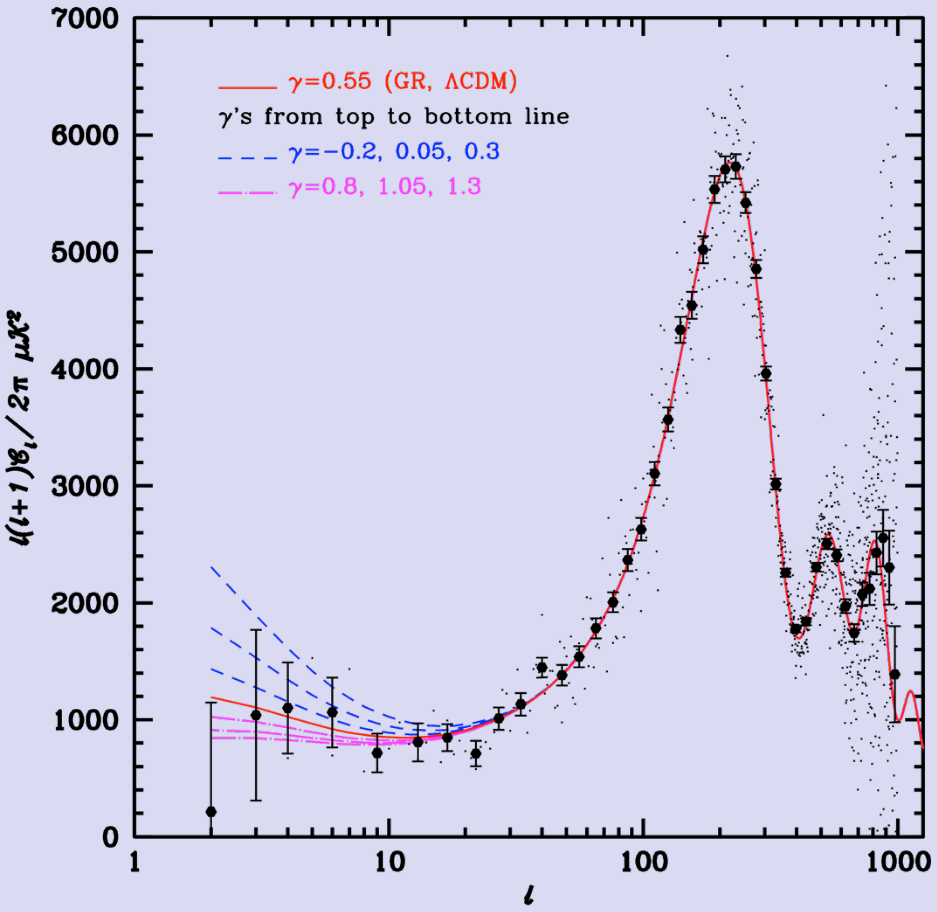
Green, dotted-dashed line:
XLF alone

Red, dashed line:
SNIa+fgas+BAO+CMB(ISW)

Blue, solid line:
XLF+SNIa+fgas+BAO+CMB(ISW)

Integrated Sachs-Wolfe effect

Rapetti et al 08



The ISW effect changes for different growth rates.

$$\Delta_l^{\rm ISW}(k) = 2\int dt\, {\rm e}^{-\tau(t)}\phi' j_l\left[k(t-t_0)\right]$$

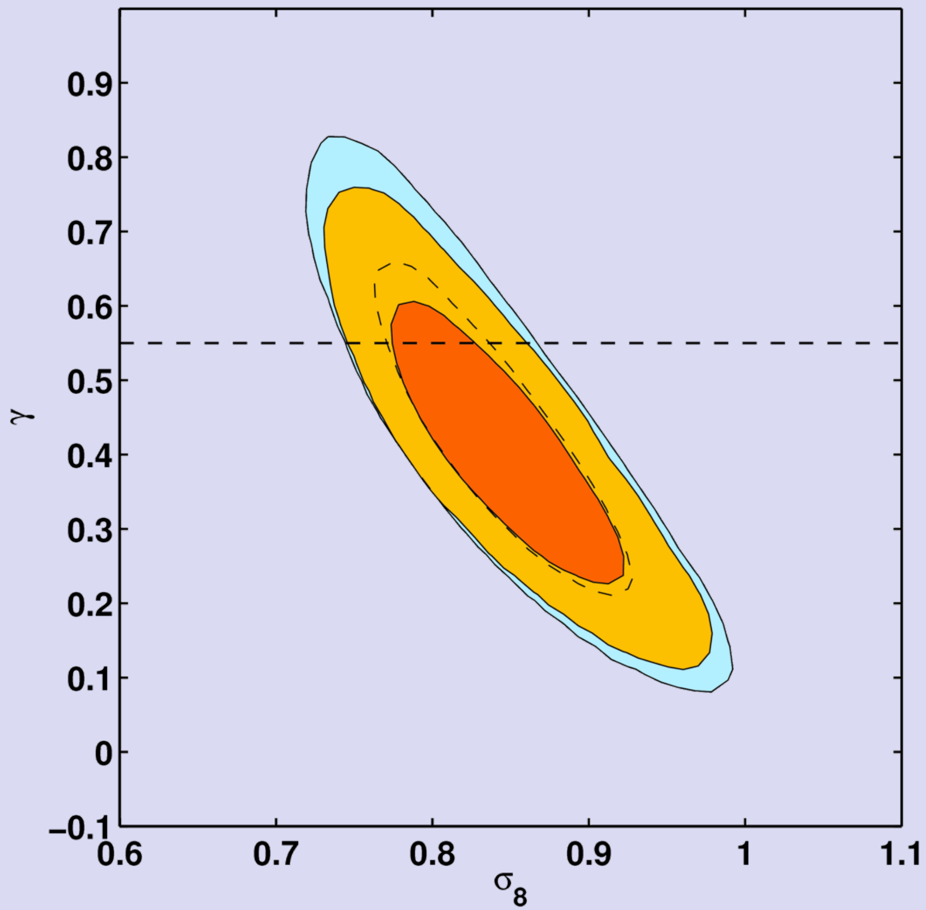
$$\phi' = -\frac{4\pi G}{k^2}\frac{\partial}{\partial t}\left(a^2\,\delta\rho_{\rm m}\right)$$

$$\frac{d\delta}{da} = \frac{\delta}{a}\Omega_{\rm m}(a)^{\gamma}$$

We consistently use it, but it is not competitive with XLF in constraining γ

Flat ΛCDM + growth index γ

Rapetti et al 10



XLF: BCS+REFLEX+MACS (z<0.5)
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cluster f_{gas} (Allen et al 2008)

For General Relativity γ~0.55

Gold: Self-similar evolution and constant scatter
Blue: Marginalizing over β^{lm}_2 and σ'_{lm}

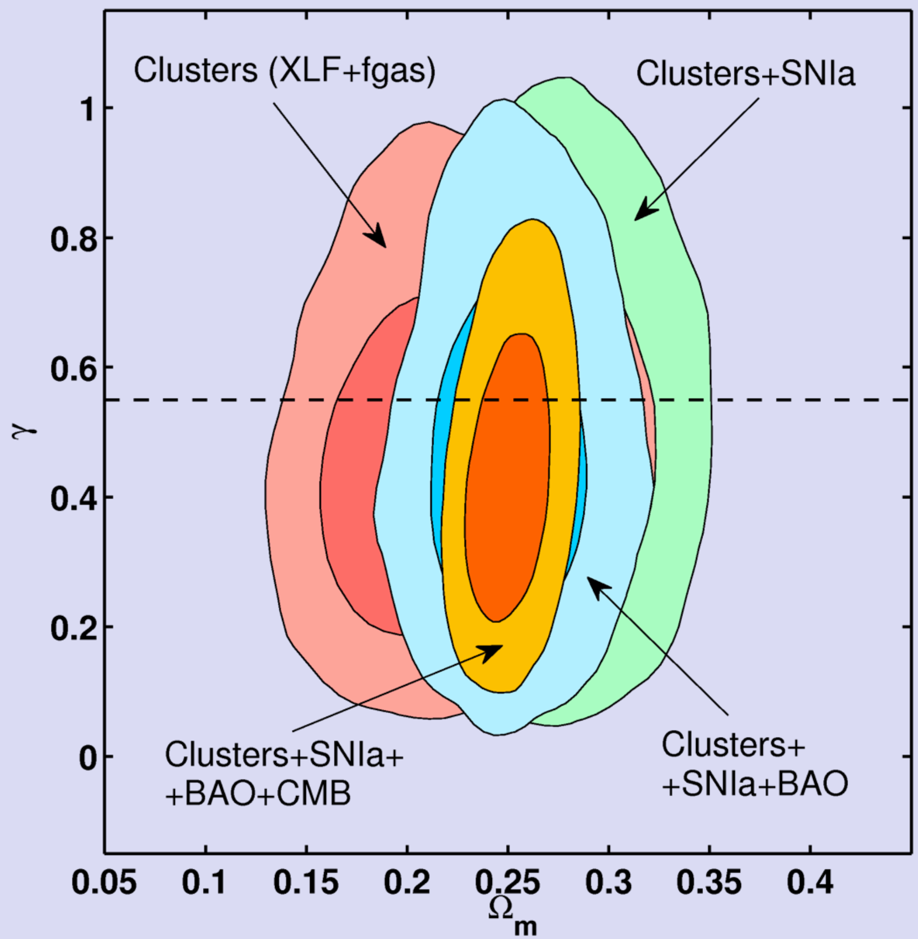
$$\gamma\left(\frac{\sigma_8}{0.8}\right)^{6.8} = 0.55^{+0.13}_{-0.10}$$

Tight correlation between σ_8 and γ:

$$\rho = -0.87$$

The impacts of the different data sets

Rapetti et al 10



Red: clusters (XLF+fgas)

Green: clusters+SNIa

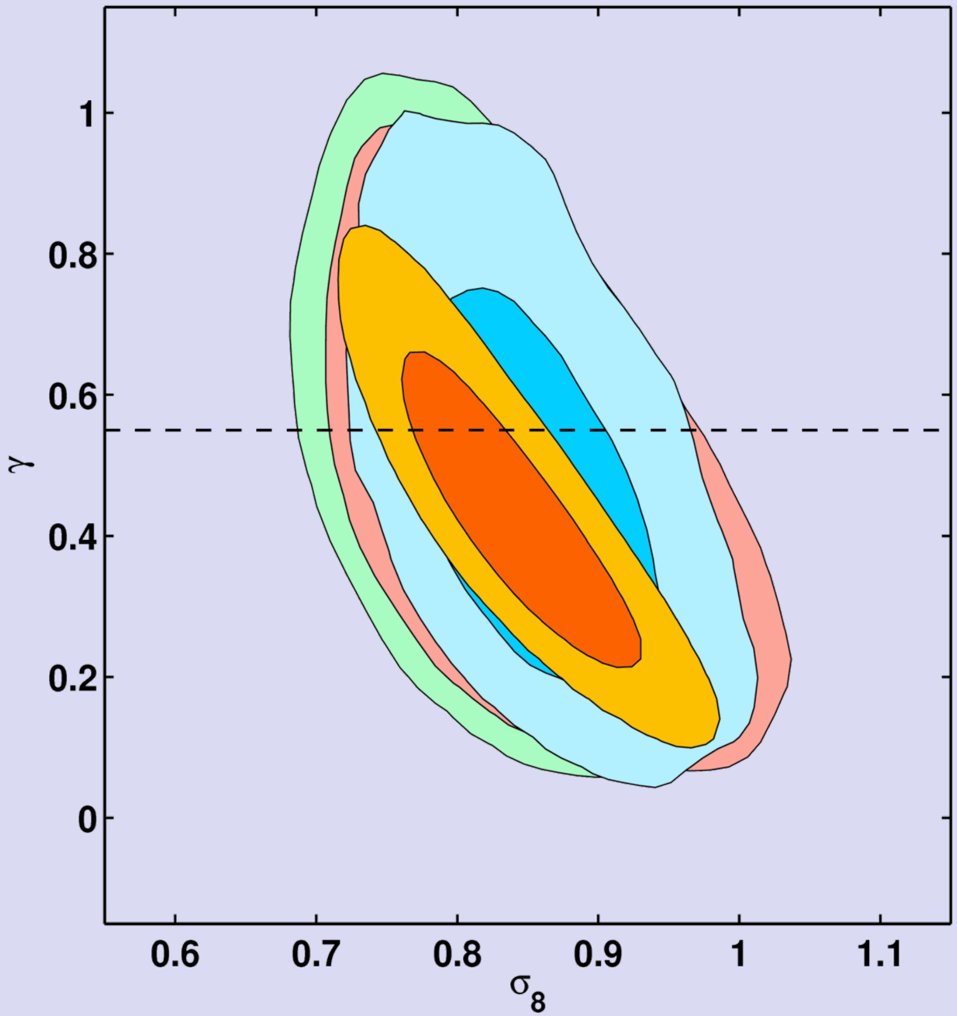
Blue: clusters+SNIa+BAO

Gold: clusters+SNIa+BAO+CMB

Adding the CMB tightens Ω_m, however the correlation with γ is weak.

The impacts of the different data sets

Rapetti et al 10



Red: clusters (XLF+fgas)

Green: clusters+SNIa

Blue: clusters+SNIa+BAO

Gold: clusters+SNIa+BAO+CMB

Adding the CMB leads to a tight correlation between σ_8 and γ thanks to the constraints on several cosmological parameters:

$$\gamma\left(\frac{\sigma_8}{0.8}\right)^{6.8} = 0.55^{+0.13}_{-0.10}$$

Strong correlation between σ_8 and γ:

$$\rho = -0.87$$

Redshift space distortions and Alcock-Paczynski effect

e.g. Blake et al 11; Beutler et al 2012; Reid et al 12

Anisotropic galaxy clustering: RSD and AP effect

Sources of anisotropy in the distribution of galaxies (2-point statistics) used to constrain the cosmological model:

- Redshift space distortions: due to velocity patterns of galaxies infalling into gravitational potential wells

$$\boxed{f\sigma_8(z)}$$

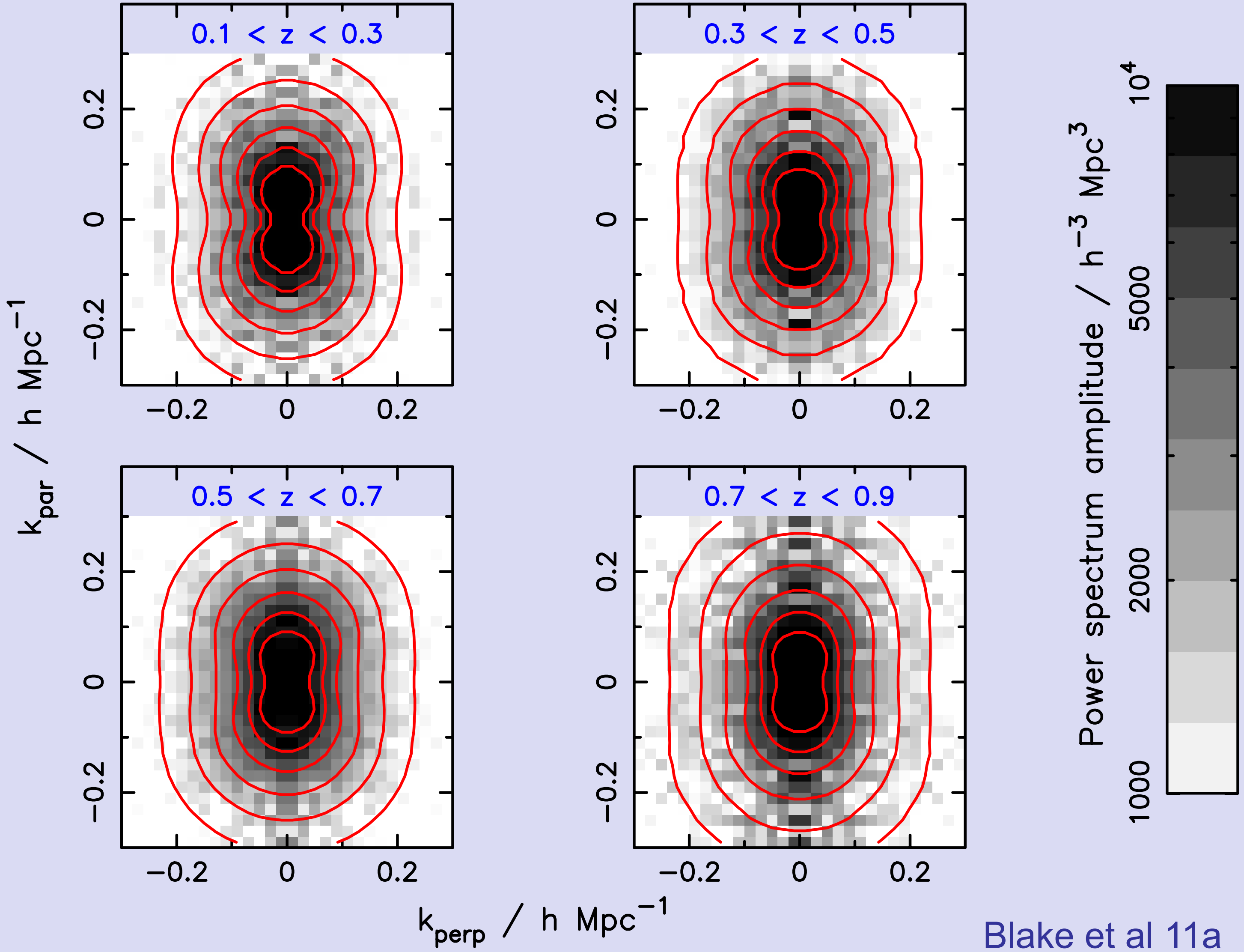
f(z) is the linear growth rate and σ_8(z) the variance in the density field at 8h^{-1}Mpc

- Alcock-Paczynski distortion: between the tangential and radial dimensions of objects or patterns when the correct cosmological model is assumed to be isotropic

$$\boxed{F(z) \;=\; (1+z)D_{\mathrm{A}}(z)H(z)/c}$$

D_A(z) is the angular diameter distance and H(z)=H_0E(z) is the Hubble parameter

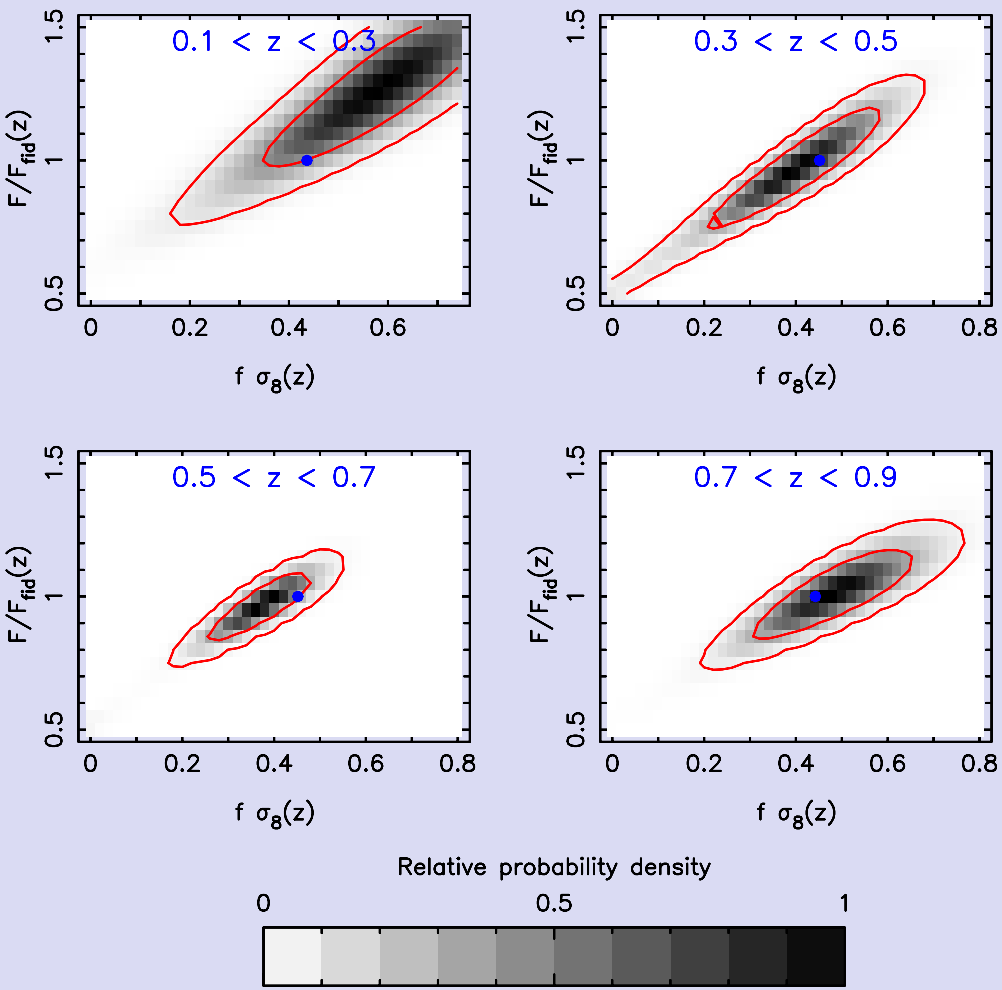
WiggleZ: two-dimensional power spectra



Blake et al 11a

WiggleZ and 6dFGS constraints on RSD and AP effect

Blake et al 11



For WiggleZ (Blake et al 11):
- We use a bivariate Gaussian likelihood on $f\sigma_8(z)$ and $F(z)$ (good approximation):

z = (0.22, 0.41, 0.60, 0.78)

$f\sigma_8(z)$ = (0.53±0.14, 0.40±0.13, 0.37±0.08, 0.49±0.12)

F(z) = (0.28±0.04, 0.44±0.07, 0.68±0.06, 0.97±0.12)

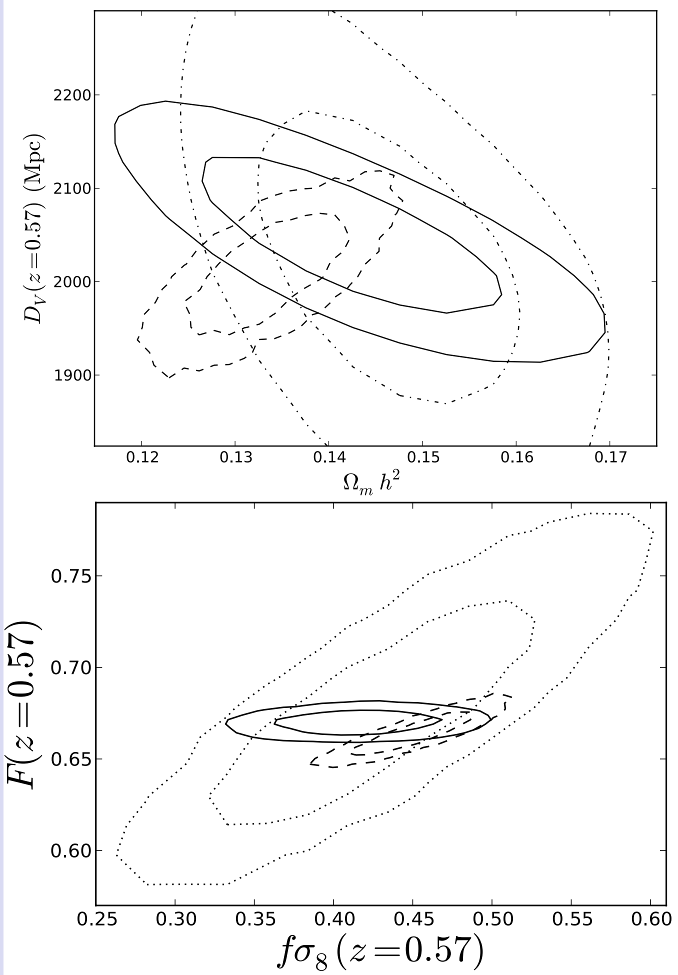
r=(0.83, 0.94, 0.89, 0.84)

For 6dFGS (Beutler et al 2012):
- We use a Gaussian likelihood on $f\sigma_8(z)$ only (since at low-z the AP effect is negligible):

$f\sigma_8(z=0.067)$ = 0.423±0.055

SDSS-III CMASS BOSS constraints

Reid et al 12



For CMASS BOSS (Reid et al 2012):

- We use either a bivariate (growth) or a trivariate (BAO) Gaussian likelihood on $f\sigma_8(z)$, F(z) and A(z) (good approximation):

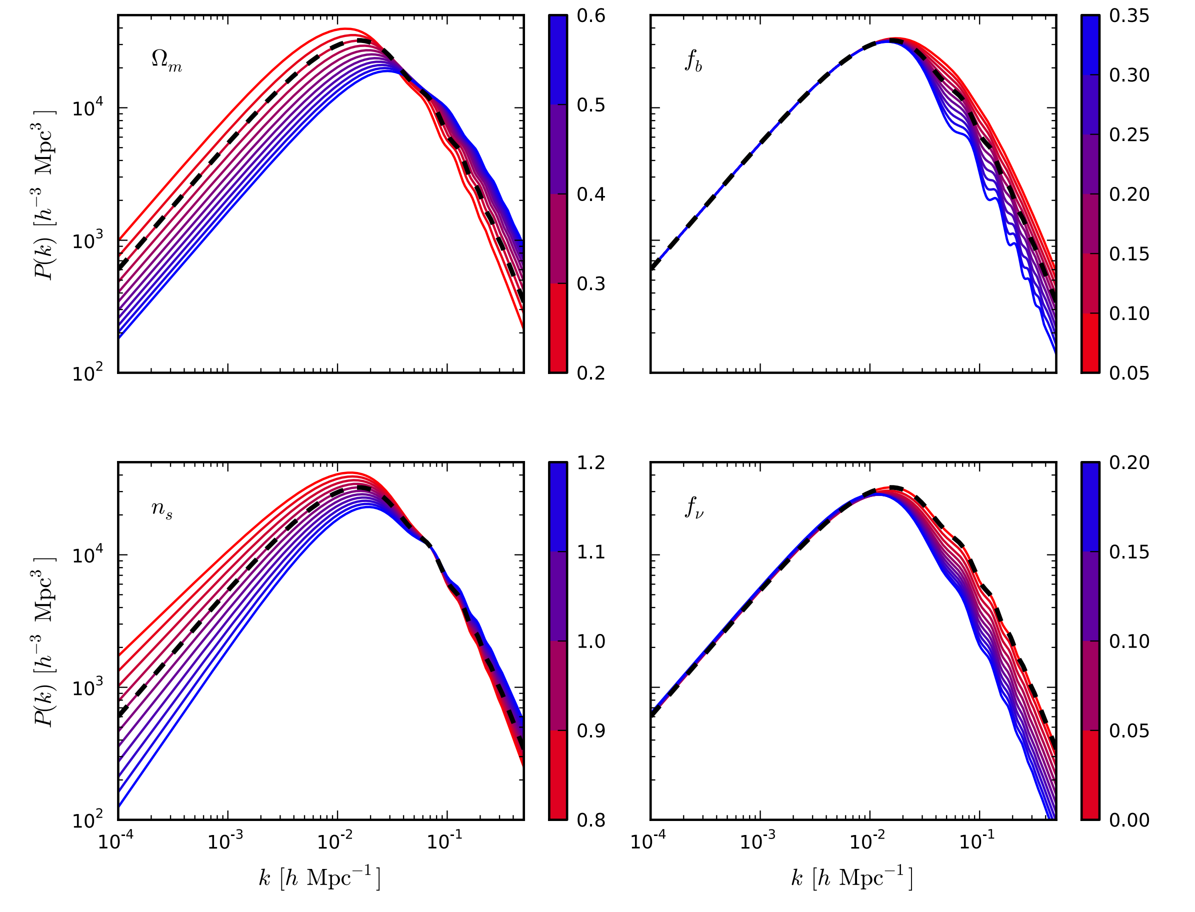
$f\sigma_8(z=0.57) = 0.43 \pm 0.07$
$F(z=0.57) = 0.68 \pm 0.04$
$A(z=0.57) = 1.023 \pm 0.019$

$r_{f\sigma F} = 0.87$
$r_{f\sigma A} = -0.0086$
$r_{FA} = -0.080$

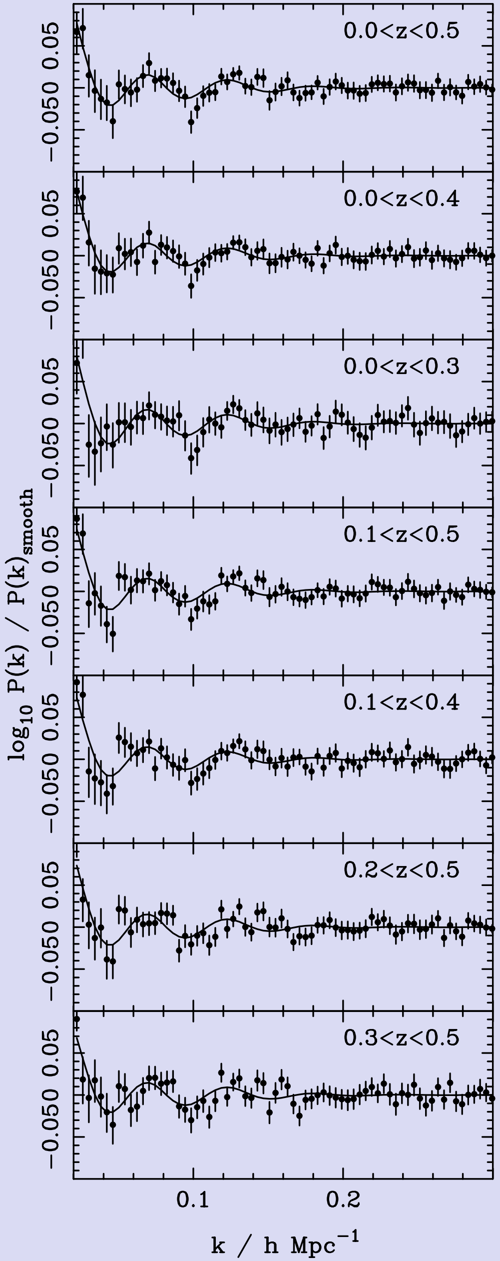
$$A(z) \equiv (D_{\rm v}/r_{\rm s})/(D_{\rm v}/r_{\rm s})_{\rm fiducial}$$
$$D_V(z) = [(1+z)^2 D_A(z)^2 cz/H(z)]^{1/3}$$

Large scale distributions of galaxies: matter power spectrum

Parkinson et al 12, WiggleZ final release



Percival et al 10, SDSS DR7



WiggleZ CosmoMC module: http://smp.uq.edu.au/wigglez-data

Combined constraints on growth and expansion: breaking degeneracies

"A combined measurement of cosmic growth and expansion from clusters of galaxies, the CMB and galaxy clustering",

arXiv:1205.4679

David Rapetti, Chris Blake, Steven Allen, Adam Mantz, David Parkinson, Florian Beutler

Modeling the abundance of clusters and their scaling relations

$$n(M,z) = \int_0^M \mathcal{F}(\sigma,z)\,\frac{\rho_{\rm m}}{M'}\,\frac{d\ln\sigma^{-1}}{dM'}\,dM'$$

Number density of dark matter halos

$$\mathcal{F}(\sigma,z) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right] e^{-c/\sigma^2}$$

Fitting formulae from N-body simulations

$$x(z) = x_0(1+z)^{\varepsilon\alpha_{\rm x}}$$

x being A, a, b, or c (Tinker et al 2008)

$$\langle \ell(m)\rangle = \beta_0^{\ell m} + \beta_1^{\ell m} m + \beta_2^{\ell m}\log_{10}(1+z)$$

Luminosity-mass relation

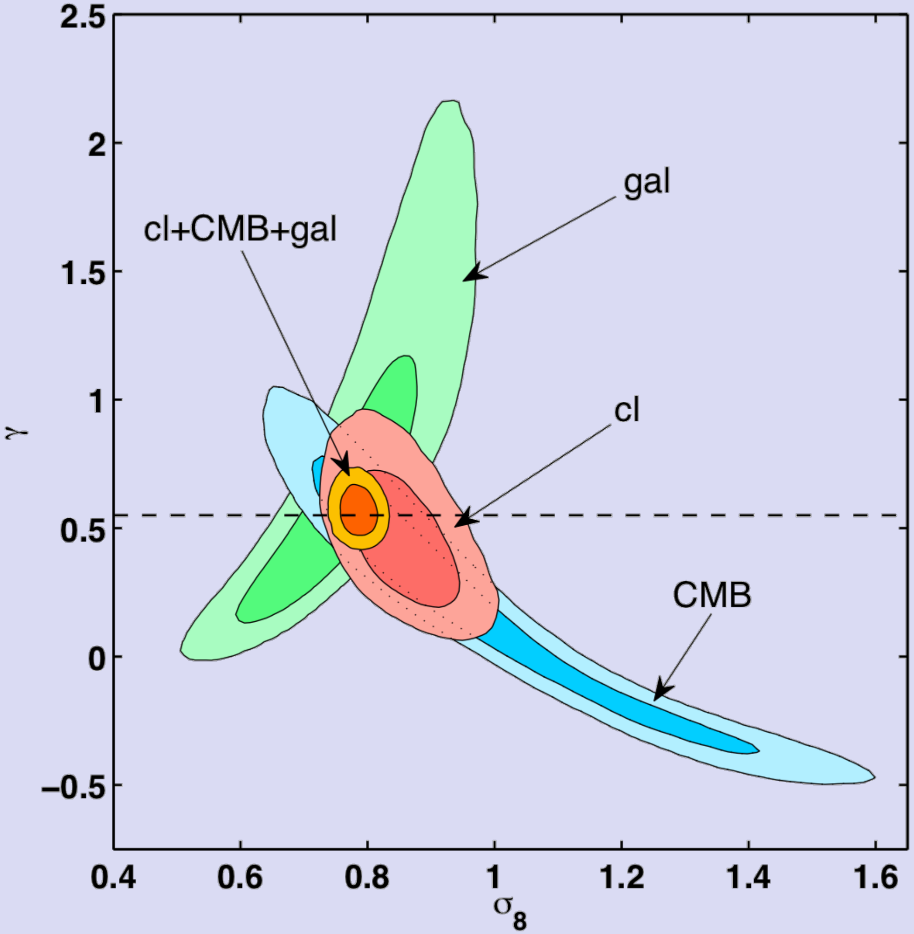
$$\sigma_{\ell m}(z) = \sigma_{\ell m}(1+\sigma'_{\ell m} z)$$

Scatter in the luminosity-mass relation

(same expressions for the temperature-mass relation but changing l for t)

Flat ΛCDM + growth index γ

Rapetti et al 12



clusters (XLF+f_{gas}): BCS+REFLEX +MACS

CMB (ISW): WMAP

galaxies (RSD+AP): WiggleZ +6dFGS+BOSS

Gold: clusters+CMB+galaxies

(+BAO+SNIa+SH0ES)

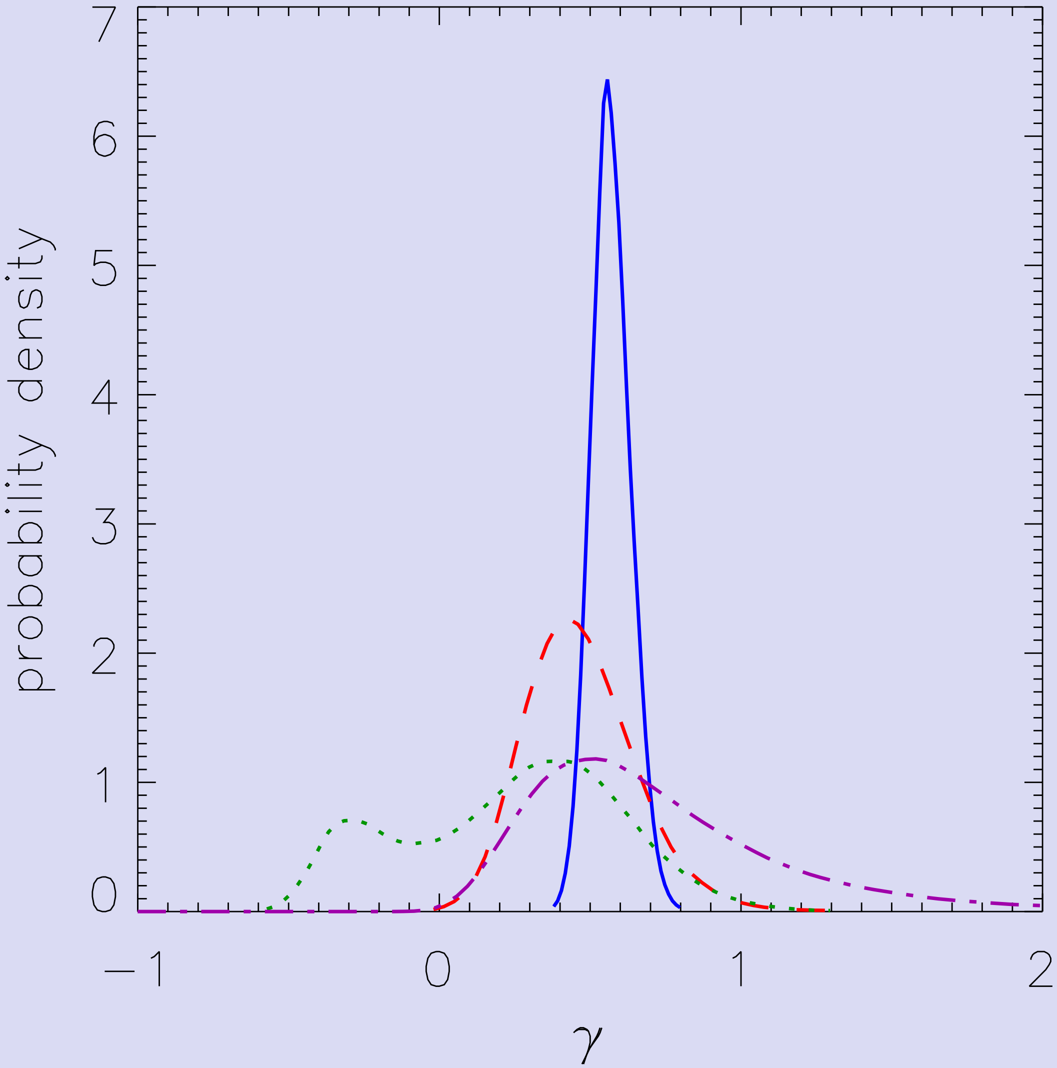
$$\gamma = 0.576^{+0.058}_{-0.059}$$

$$\sigma_8 = 0.789 \pm 0.019$$

$$\Omega_m = 0.255 \pm 0.011$$

$$H_0 = 72.1 \pm 1.0$$

Flat ΛCDM + growth index γ



Green, dotted line:
CMB alone

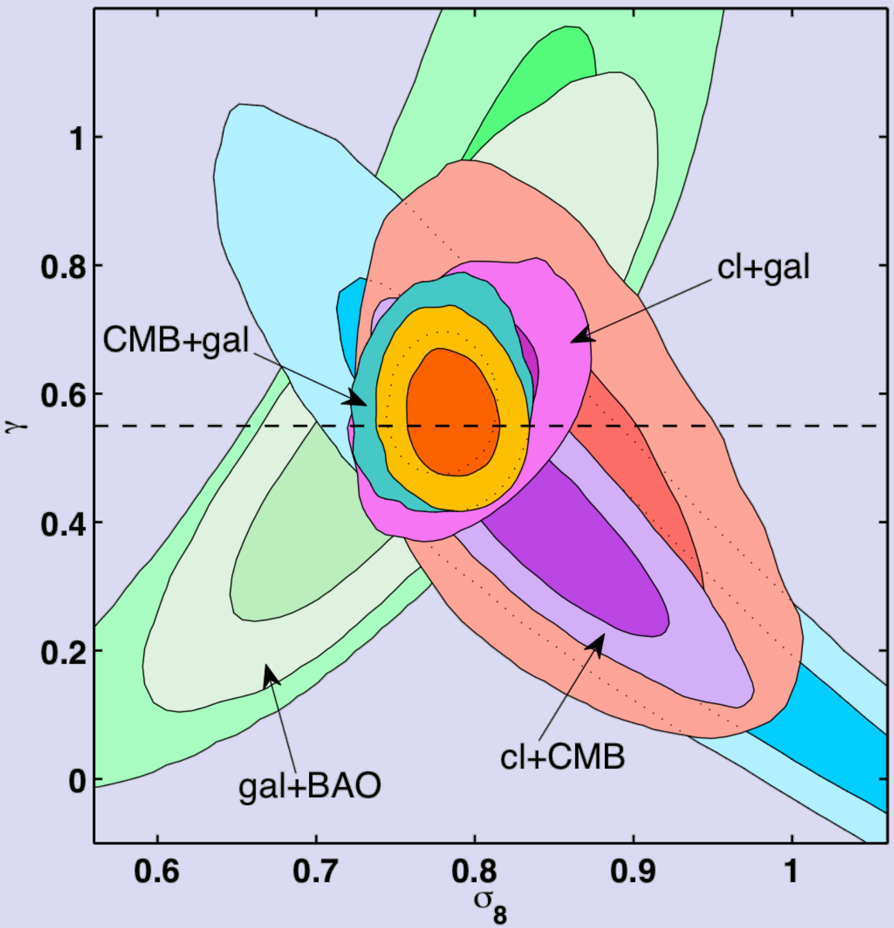
Magenta, dotted-dashed line:
galaxies

Red, dashed line:
clusters

Blue, solid line:
clusters+CMB(ISW)+galaxies

Flat ΛCDM + growth index γ

Rapetti et al 12



clusters (XLF+f_{gas}): BCS+REFLEX +MACS

CMB (ISW): WMAP

galaxies (RSD+AP): WiggleZ +6dFGS+BOSS

For General Relativity γ~0.55

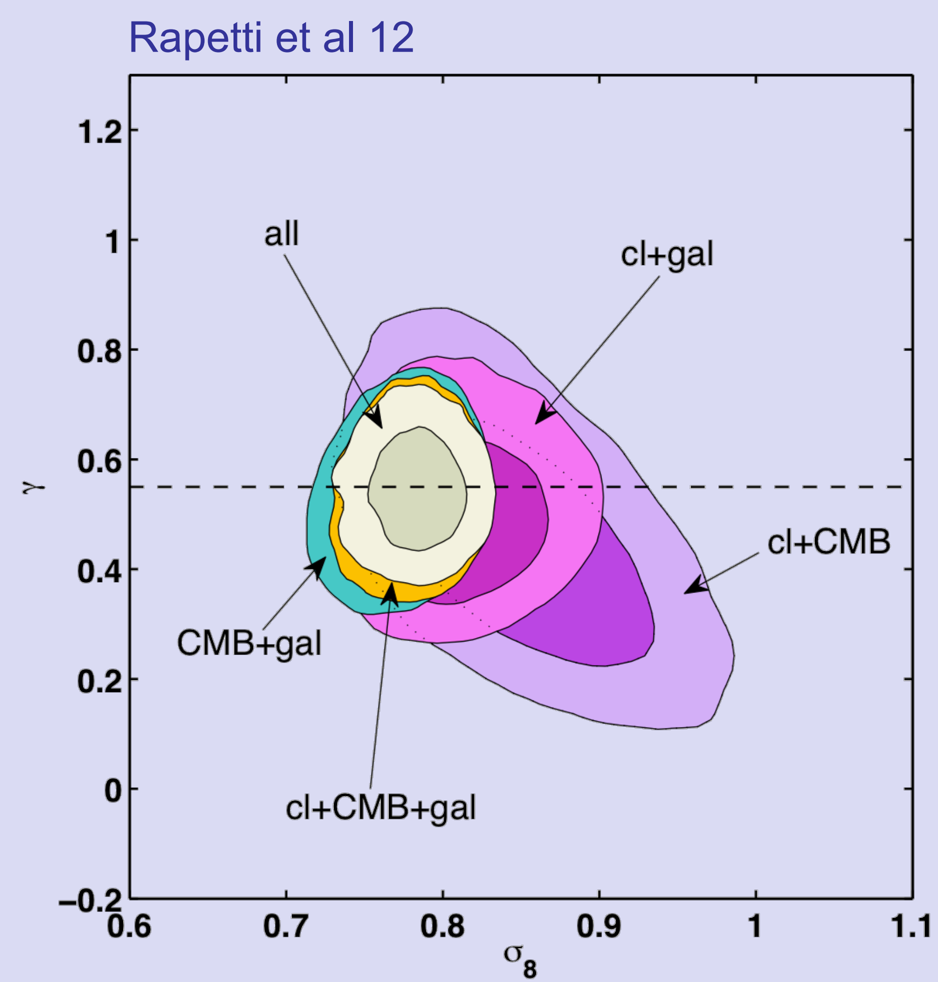
Magenta: clusters+galaxies

Purple: clusters+CMB

Turquoise: CMB+galaxies

Gold: clusters+CMB+galaxies

Flat wCDM + growth index γ: growth plane



For General Relativity γ~0.55

Magenta: clusters+galaxies

Purple: clusters+CMB

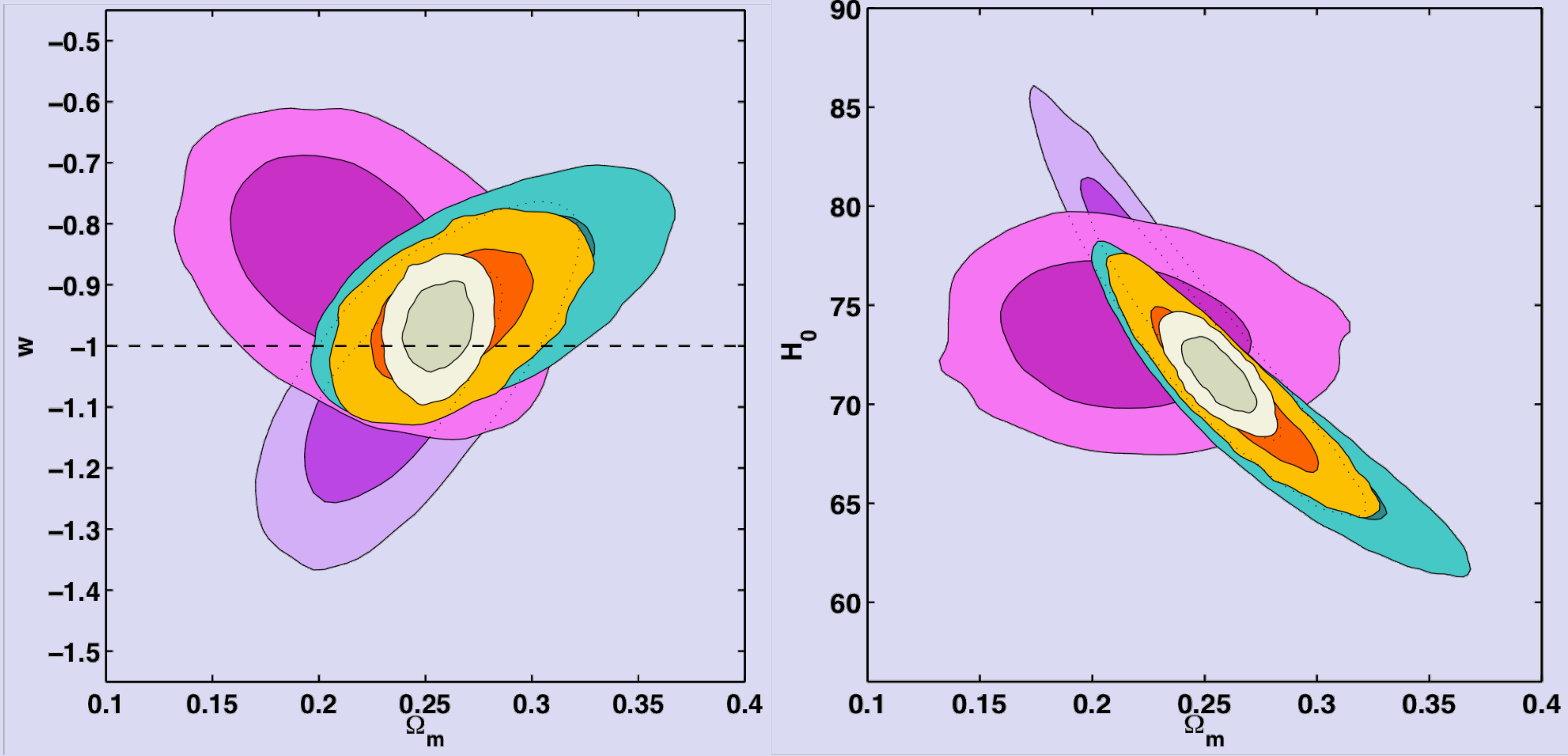
Turquoise: CMB+galaxies

Gold: clusters+CMB+galaxies

Platinum: clusters+CMB+galaxies +BAO (Reid et al 12; Percival et al 10)+SNIa (Suzuki et al 12) +SH0ES (Riess et al 11)

Flat wCDM + growth index γ: expansion planes

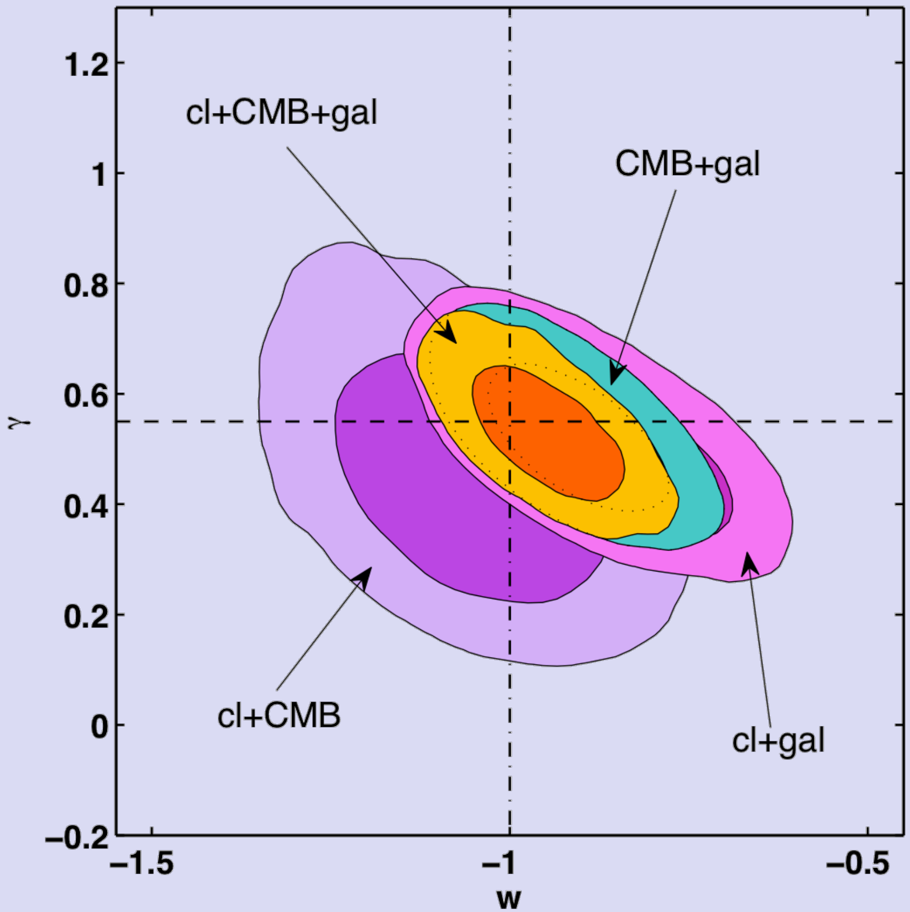
Rapetti et al 12



Platinum: clusters + CMB + galaxies + BAO (Reid et al 12; Percival et al 10) + SNIa (Suzuki et al 12) + SH0ES (Riess et al 11)

Flat wCDM + growth index γ: growth+expansion

Rapetti et al 12



For General Relativity γ~0.55

Magenta: clusters+galaxies

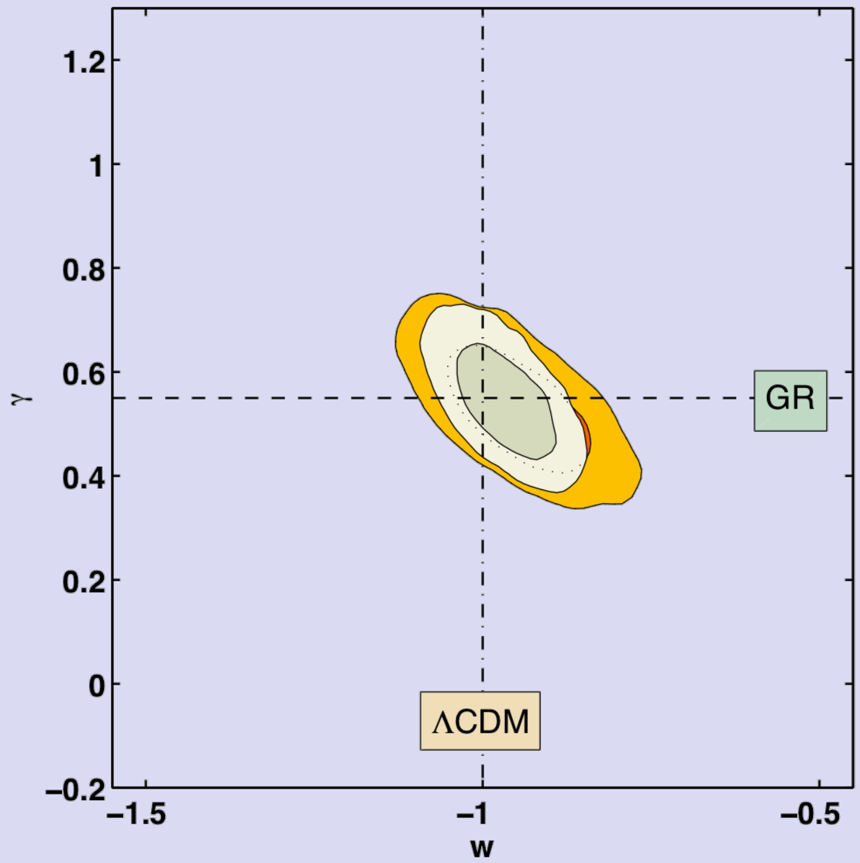
Purple: clusters+CMB

Turquoise: CMB+galaxies

Gold: clusters+CMB+galaxies

Flat wCDM + growth index γ: growth+expansion

Rapetti et al 12



For General Relativity γ~0.55

For ΛCDM w=-1

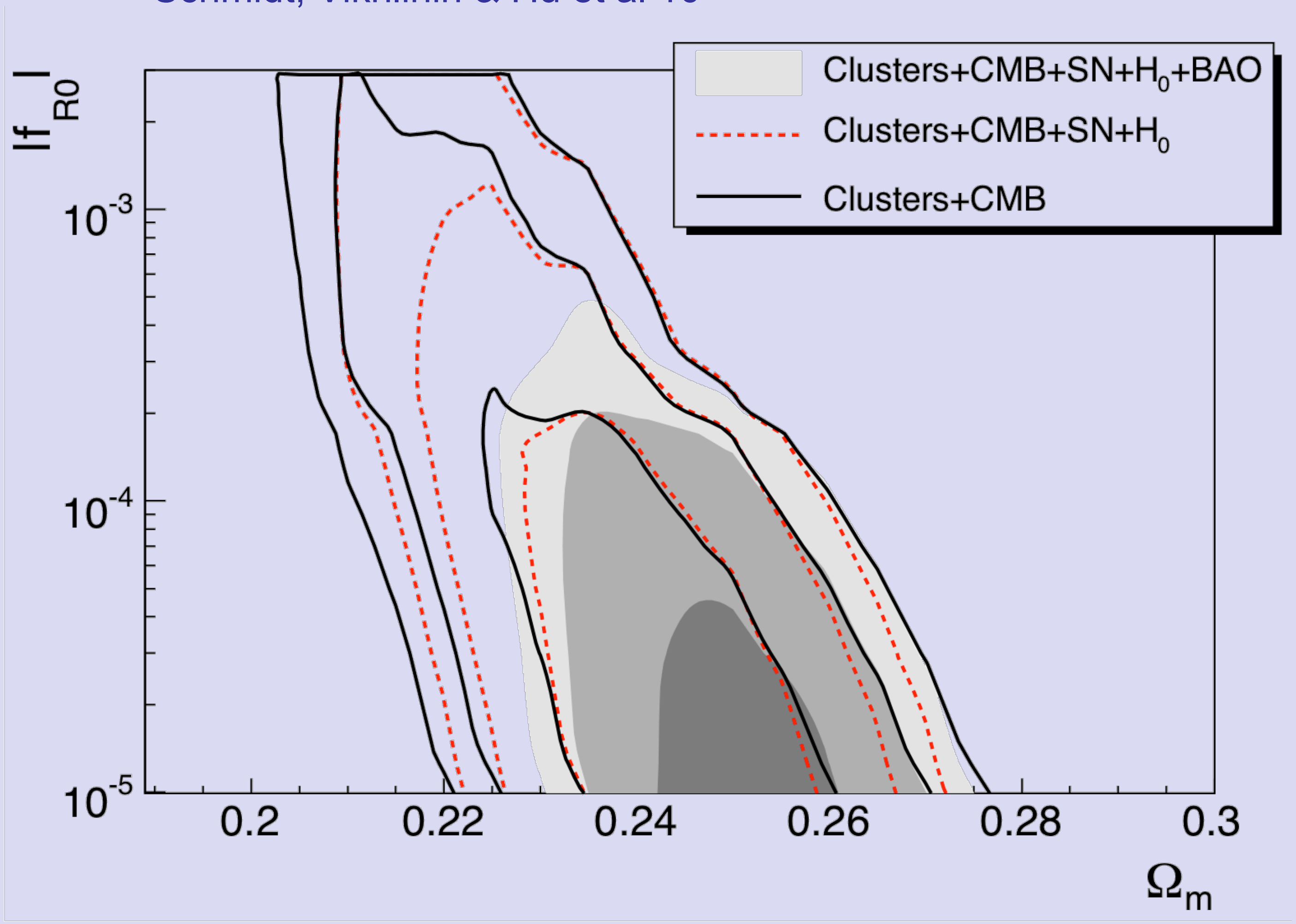
Gold: clusters+CMB+galaxies

Platinum: clusters+CMB+galaxies +BAO+SNIa+SH0ES

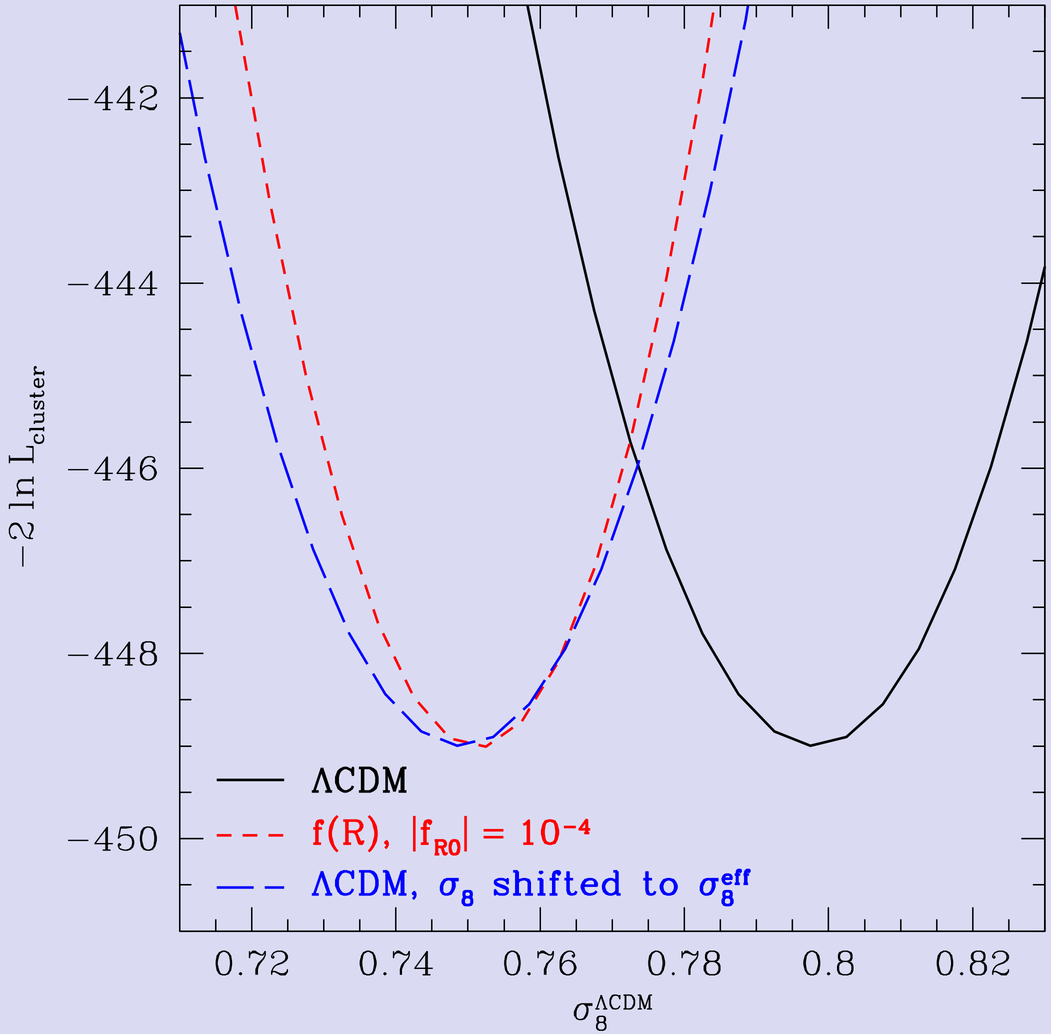
$$\gamma = 0.546^{+0.071}_{-0.072}$$
$$\sigma_8 = 0.783^{+0.020}_{-0.019}$$
$$w = -0.968 \pm 0.049$$
$$\Omega_m = 0.256 \pm 0.011$$
$$H_0 = 71.5 \pm 1.3$$

flat+ΛCDM expansion history, f(R) gravity model

Schmidt, Vikhlinin & Hu et al 10



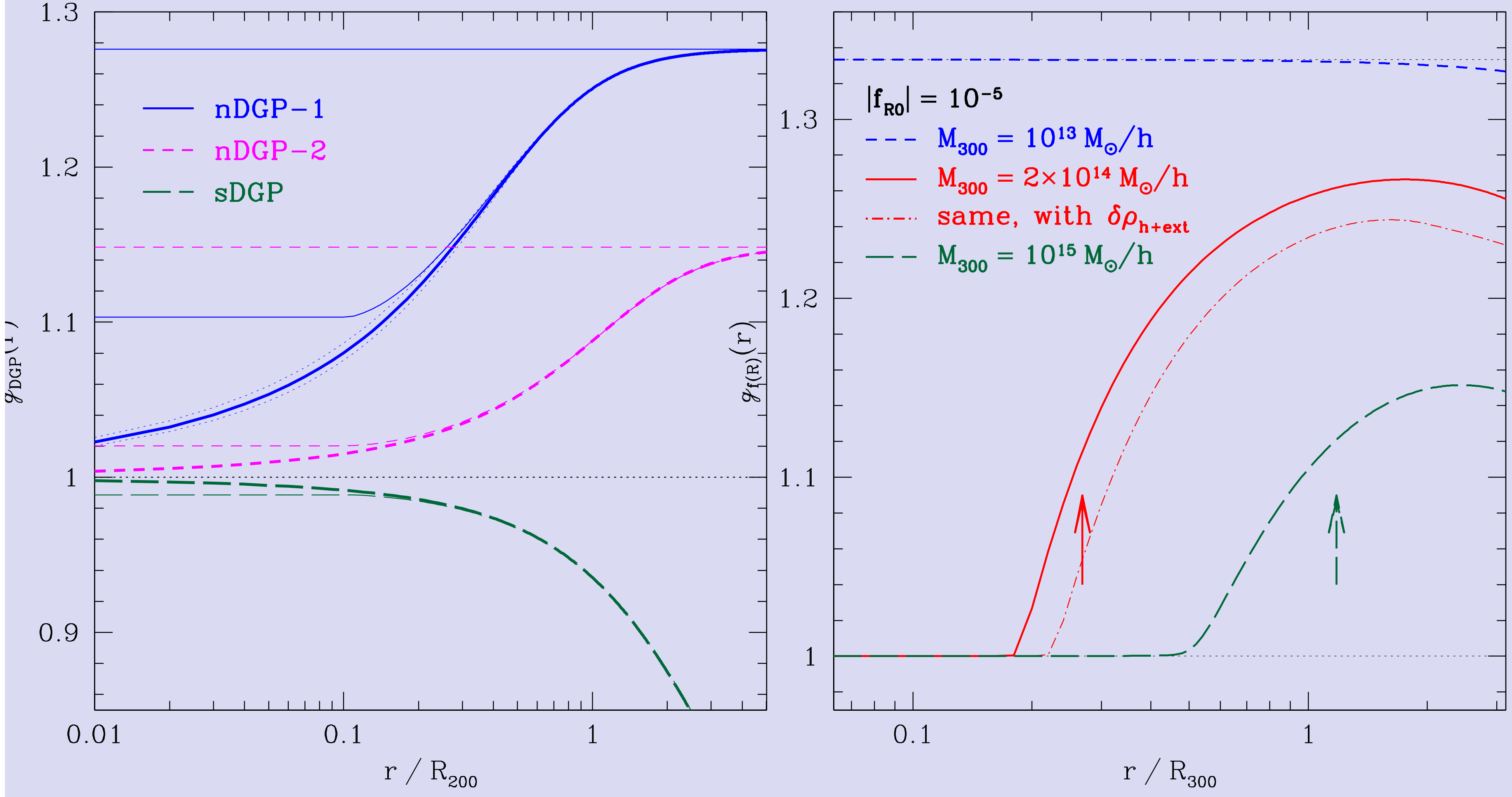
flat+ΛCDM expansion history, f(R) gravity model



Schmidt, Vikhlinin & Hu et al 10

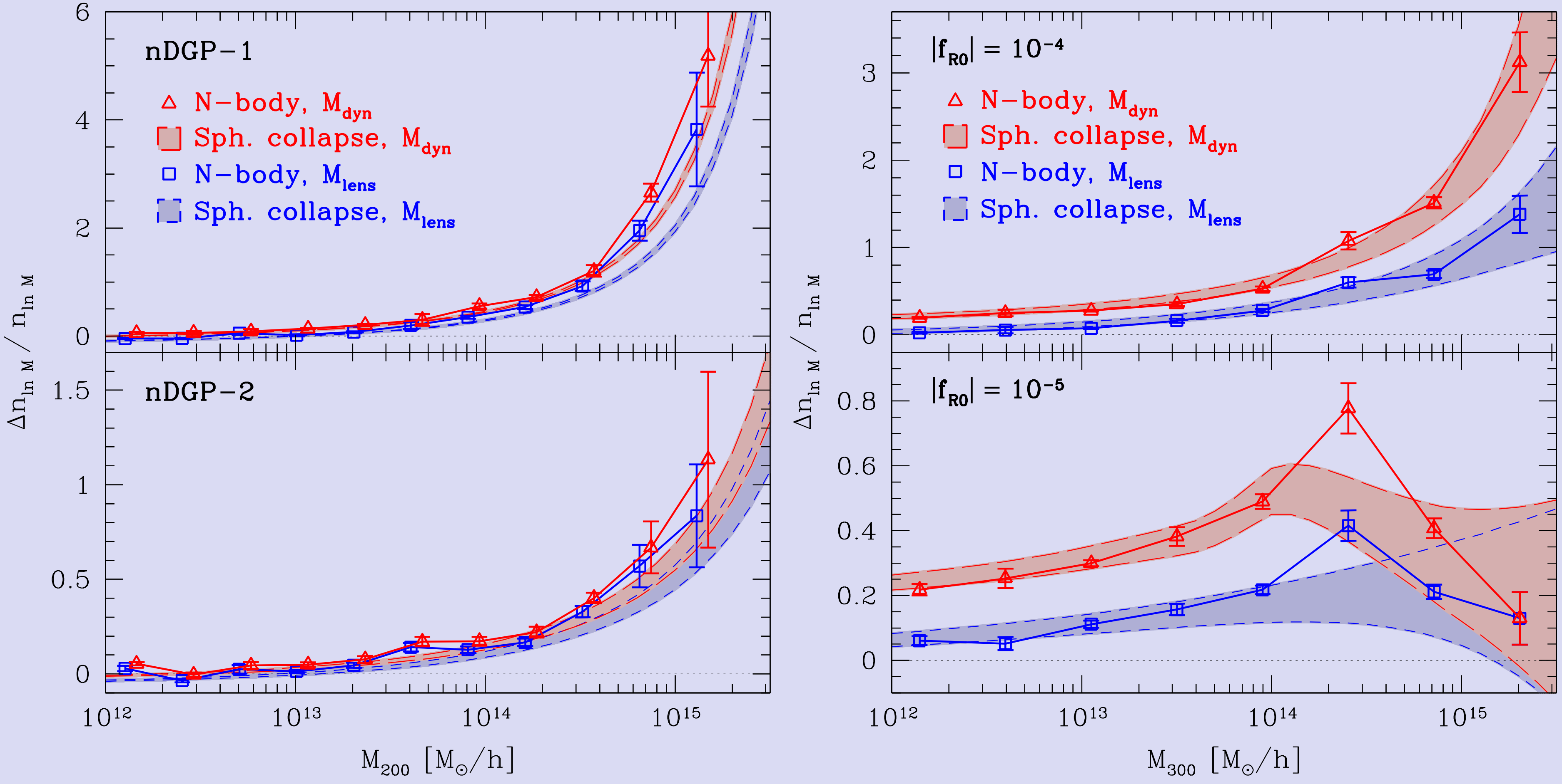
Cluster mass profiles in DGP and f(R)

Schmidt 10



Cluster mass profiles in DGP and f(R)

Schmidt 10



Summary

- We have performed a consistency test of General Relativity (growth rate) at large scales using cluster growth data: BCS+REFLEX+Bright MACS, Tinker et al 2008 mass function, 94 clusters with X-ray follow-up observations as well as other cosmological data from f_{gas}+SNIa+CMB+BAO.

- We obtain a tight correlation $\gamma(\sigma_8/0.8)^{6.8}$=0.55+0.13-0.10 for the flat ΛCDM model. This promises significant improvements on γ by adding independent constraints on σ_8.

- Our results are robust when allowing additional evolution in the luminosity-mass relation and its scatter thanks to the wide redshift range covered by the follow-up data.

- Simultaneously fitting γ and w, we find that current data is consistent with GR +ΛCDM.

Remember: choose a dark energy model to implement in CosmoMC tomorrow

- Some recent dark energy reviews:
  - Copeland, Sami, Tsujikawa, 06, Int. J. Mod Phys D
  - Frieman, Turner, Huterer, 08, Ann. Rev. Astr. & Astrophys., 46, 385
  - Weinberg, Mortonson, Eisenstein, Hirata, Riess, Rozo, 12, for Phys. Reports, arXiv:1201.2434
- Dark energy task forces and future dark energy missions:
  - Albrecht, Bernstein, Cahn, Freedman, Hewitt, Hu, Huth, Kamionkowski, Kolb, Knox, Mather, Staggs, Suntzeff, 06, arXiv/0609591
  - Albrecht, Amendola, Bernstein, Clowe, Eisenstein, Guzzo, Hirata, Huterer, Kirshner, Kolb, Nichol, 09, arXiv:0901.0721
  - Amendola, et al (Euclid Satellite), 12, arXiv:1206.1225

Modern cosmology with X-ray luminous clusters of galaxies

Friday Lecture/Practice: Implementing and constraining a theoretical model using CosmoMC

David Rapetti

DARK Fellow

Dark Cosmology Centre, Niels Bohr Institute

Dark Cosmology Centre

University of Copenhagen

Other public cosmological codes

- CLASS: http://lesgourg.web.cern.ch/lesgourg/class.php
(Blas, Lesgourgues, Tram, 11, JCAP, 07, 034)

- Analyse this! and CMBEASY: http://www.thphys.uni-heidelberg.de/~robbers/cmbeasy/
(Doran & Müller, 04, JCAP, 09, 003; Doran, 05, JCAP, 10, 011)

- CosmoPMC: http://www2.iap.fr/users/kilbinge/CosmoPMC/
(Kilbinger et al, 11, arXiv:1101.0950)

- CosmoNest: as add-on for CosmoMC: http://cosmonest.org/
(Mukherjee, Parkinson, Liddle, 06, ApJ, 638, 51)

- MultiNest: bayesian inference: http://ccpforge.cse.rl.ac.uk/gf/project/multinest/
(Feroz, Hobson, Bridges, 09, MNRAS, 398, 1601)

Finish previous exercises

Exercise: Use the SNe Ia code of the Union 2.1 in CosmoMC to obtain the constraints on the paper (Suzuki et al 12). You can also use the SCP website http://supernova.lbl.gov/Union/.

Exercise: Plot the data from the data folder in CosmoMC (with the corresponding error bars) to get familiar with it.

Exercise: Using the fgas module for CosmoMC, reproduce the constraints in Allen et al 08 for the non-flat LCDM model and flat wCDM models; obtain the 2D (or 1D) marginalized constraints.

Exercise:Test the robustness of the previous results by sensibly changing the allowances in the fgas module and obtaining new constraints.

CosmoMC project

- Implement your chosen theoretical model

- To modify the expansion history model go into the camb folder and appropriately change the file equations.f90

- Include the new variables of your model into cosmomc

- For this, modify accordingly files in the source folder such as driver.f90, etc. (hint: you can trace other equivalent parameters to see which other files you need to modify)

- Remember to include your new parameters in your params.ini

- Compile, run with your choice of expansion data sets (fgas, SNe Ia, BAO, etc.), and analyze the chains with getdist with a corresponding distparams.ini. Again, ask questions when needed and good luck!